

- 35 Solve the simultaneous equations

$$x + y = 2$$

$$3y^2 - x^2 = 12$$

(6 marks)

- 36 Solve the following equation algebraically, showing each step of your working:

$$(4^{x-1})^2 - 10(4^{x-1}) + 16 = 0$$

(5 marks)

- 37 In the triangle ABC , $AB = 14$ cm, $BC = 6$ cm and $\angle ABC = \theta$. The area of the triangle is 24 cm².

a Find the two possible values of θ correct to 1 decimal place.

(3 marks)

b Given that AC is the longest side of the triangle, find the length of AC .

(2 marks)

- 38 The expression $16 - 6x - x^2$ can be written in the form $q - (x + p)^2$, where p and q are integers.

a Find the value of p and the value of q .

(3 marks)

b Calculate the discriminant of $16 - 6x - x^2$.

(1 mark)

c Sketch the curve with equation $y = 16 - 6x - x^2$, showing clearly the coordinates of any points where the curve crosses the coordinate axes.

(3 marks)

- 39 a On the same set of axes, sketch the graphs of $y = \cos x$ and $y = \cos 3x$ for $0 \leq x \leq 360^\circ$.

(3 marks)

b Describe the transformation from the graph of $y = \cos x$ to $y = \cos 3x$.

(2 marks)

c Given that $\cos 3x = \cos 60^\circ$, use your graph to find all the solutions to the equation $\cos 3x = \cos 60^\circ$ for $0 \leq x \leq 360^\circ$.

(4 marks)

- 40 a Find the first 3 terms, in ascending powers of x , of the binomial expansion of $\left(2 - \frac{x}{15}\right)^{10}$.
Give each term in its simplest form.

(4 marks)

$f(x) = (p + qx)\left(2 - \frac{x}{15}\right)^{10}$, where p and q are constants. Given that the first two terms, in ascending powers of x , in the series expansion of $f(x)$ are 512 and $-\frac{256x}{3}$,

b find the value of p

(2 marks)

c find the value of q .

(2 marks)

- 41 $f(x) = 6x^3 + 5x^2 - 3x - 2$

a Factorise $f(x)$ completely.

(6 marks)

b Show that $\frac{6x^3 + 5x^2 - 3x - 2}{2x^3 + 3x^2 + x}$ can be written in the form $A + \frac{B}{x}$, where A and B are integers to be found.

(3 marks)

- 42 The circle C has centre $(2, -1)$ and passes through the point $P(8, 5)$.

a Find an equation for C . (4 marks)

The circle crosses the positive y -axis at the point Q .

b Find the area of the triangle OPQ , where O is the origin, giving your answer in its simplest surd form. (4 marks)

- 43 $f(x) = x^2 - 6x + 16$, $x \in \mathbb{R}$.

a Show that $f(x)$ can be written in the form $(x - p)^2 + q$, where p and q are constants to be found. (2 marks)

b Hence sketch the graph of $y = f(x)$ showing the coordinates of the turning point. (3 marks)

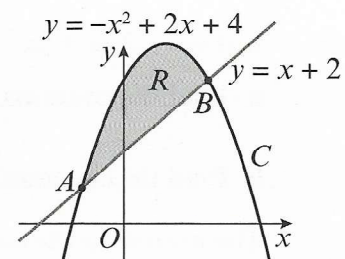
c Given that the equation $f(x) = k$, where k is a constant, has two distinct real solutions, find the range of possible values of k . (3 marks)

- 44 The diagram shows a sketch of part of the curve C with equation $y = -x^2 + 2x + 4$.

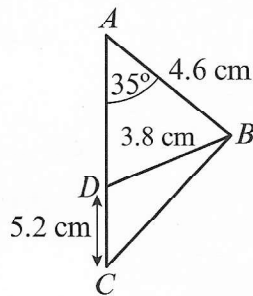
The straight line $y = x + 2$ crosses the curve at the points A and B .

a Find the coordinates of points A and B . (2 marks)

b Find the area of the finite region R . (7 marks)



- 45 The diagram shows triangle ABC .



Calculate the area of triangle ABC . (6 marks)

- 46 The path of a toy rocket launched into the air can be modelled by the equation $y = x - 0.05x^2$, where x is the horizontal distance the rocket travels (in metres) and y is the vertical height (in metres) measured from where the rocket was fired.

a Sketch the graph of $y = x - 0.05x^2$ to show the path of the rocket. (2 marks)

b State the horizontal distance travelled by the rocket when it lands on the ground. (3 marks)

In reality the rocket achieves a maximum height of 4.5 m.

c Evaluate the model in light of this information. (2 marks)

- 47 Solve $6 \tan^2 x = 11 \tan x + 10$ for $0 \leq x < 360^\circ$, giving your answers to 1 decimal place.
You must show each step of your working. (6 marks)
- 48 A curve has equation $y = x^3 + 3x^2 - 9x + 6$.
- a Find $\frac{dy}{dx}$ (2 marks)
- b Hence find the range of values of x for which y is increasing.
Write your answer using set notation. (4 marks)
- 49 a Factorise completely $36x - 25x^3$. (3 marks)
- b Sketch the curve C with equation $y = 36x - 25x^3$.
Show on your sketch the coordinates at which the curve meets the x -axis. (3 marks)
- The points A and B lie on C and have x -coordinates of -1 and 1 respectively.
- c Show that the length of AB is $2\sqrt{k}$ where k is a constant to be found. (4 marks)
- 50 $f(x) = 4 - 2\sqrt{x} + \frac{1}{2}x^2$
- a Find the coordinates of the stationary point on the curve with equation $y = f(x)$. (3 marks)
- b Find the equation of the normal to the curve at $P(4, 8)$. (5 marks)
- The normal to the curve at P intersects the x -axis at A .
- c Find the area of the triangle OAP . (3 marks)
- 51 a Fully expand $(1 + x)^5$ (3 marks)
- b Use your answer to part a to show that:
- i $(1 + \sqrt{5})^5 = 176 + 80\sqrt{5}$
- ii $\log_2(1 + \sqrt{5})^5 = p + \log_2(11 + 5\sqrt{5})$ where p is an integer. (6 marks)
- 52 $f(x) = x^2 - 6x + 13$, $x \in \mathbb{R}$.
- a Express $f(x)$ in the form $(x - a)^2 + b$ where a and b are constants. (2 marks)
- The curve C with equation $y = f(x)$ crosses the y -axis at the point P and has a minimum point at the point Q .
- b Sketch the graph of C , showing the coordinates of points P and Q . (3 marks)
- c Find the length of the line segment PQ , writing your answer as a simplified surd. (3 marks)