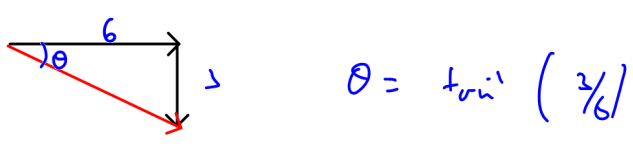


10.01

- 5 Given that $\mathbf{a} = 6\mathbf{i} - 3\mathbf{j}$, find:
- a the unit vector in the same direction of $\mathbf{a}$ (3 marks)
 - b the angle that $\mathbf{a}$ makes with the positive x -axis. (2 marks)

a) $\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$ $|\mathbf{a}| = \sqrt{6^2 + 3^2} = 3\sqrt{5}$

$\hat{\mathbf{a}} = \frac{1}{3\sqrt{5}}(6\mathbf{i} + 3\mathbf{j}) \Rightarrow \hat{\mathbf{a}} = \frac{2}{\sqrt{5}}\mathbf{i} - \frac{1}{\sqrt{5}}\mathbf{j}$



- 7 Show that $\frac{2\sin^2 x - 1}{\cos^2 x} \equiv \tan^2 x - 1$. (3 marks)

LHS: $\frac{2\sin^2 x - (\sin^2 x + \cos^2 x)}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x}$

$= \tan^2 x - 1 \equiv \text{RHS}$

- 9 In the binomial expansion of $(1 + 3x)^n$, where $n > 0$, the coefficient of x^2 is 6 times the coefficient of x . Work out the value of n . (4 marks)

$(1 + 3x)^n = 1 + \binom{n}{1} 3x + \binom{n}{2} (3x)^2 + \dots$

$= 1 + 3nx + \frac{n!}{2!(n-2)!} 9x^2 + \dots$

$= 1 + 3nx + \frac{n(n-1)\cancel{(n-2)!}}{2\cancel{(n-2)!}} 9x^2 + \dots$

$= 1 + 3nx + \frac{9n^2 - 9n}{2} x^2 + \dots$

$\frac{9n^2 - 9n}{2} = 6 \times 3n \Rightarrow \frac{n^2 - n}{2} = 2n$

$n^2 - n = 4n \Rightarrow n^2 - 5n = 0$

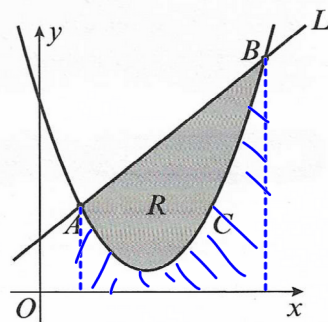
$n(n-5) = 0$

$n \neq 0 \Rightarrow n = 5$

- 10 The diagram shows part of curve C with equation $y = x^2 - 8x + 18$ and part of the line L with equation $y = 2x + 5$.

a Using an appropriate algebraic method, find the coordinates of A and B . (4 marks)

b Define the shaded region R . (2 marks)



$$a) 2x + 5 = x^2 - 8x + 18 \Rightarrow x^2 - 10x + 13 = 0$$

$$x = 5 \pm 2\sqrt{3}$$

$$x = 5 - 2\sqrt{3}$$

$$A(5 - 2\sqrt{3}, 15 - 4\sqrt{3})$$

$$B(5 + 2\sqrt{3}, 15 + 4\sqrt{3})$$

11 $f(x) = 8x^2 - \frac{80}{3}x^{\frac{3}{2}} + 5x; x \geq 0$

Find the x -coordinates of the points on the curve with equation $y = f(x)$ where the gradient is equal to -4 . (5 marks)

$$f'(x) = 16x - 40x^{\frac{1}{2}} + 5$$

$$16x - 40x^{\frac{1}{2}} + 5 = -4$$

$$16x - 40x^{\frac{1}{2}} + 9 = 0$$

$$a = x^{\frac{1}{2}}$$

$$16a^2 - 40a + 9 = 0$$

$$(4a - 9)(4a - 1) = 0$$

$$a = \frac{9}{4}$$

$$x^{\frac{1}{2}} = \frac{9}{4} \Rightarrow$$

$$x = \frac{81}{16}$$

$$a = \frac{1}{4}$$

$$x^{\frac{1}{2}} = \frac{1}{4}$$

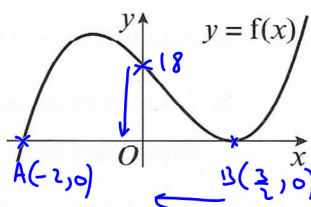
$$x = \frac{1}{16}$$

- 12 The diagram shows a sketch of part of the curve $y = f(x)$, $x \in \mathbb{R}$, where $f(x) = (2x - 3)^2(x + 2)$.

a i Given that the curve with equation $y = f(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant k .

ii Given that the curve with equation $y = f(x + c)$, $x \in \mathbb{R}$, has a local minimum point at the origin, find the value of the constant c . (3 marks)

b Show that $f'(x) = 12x^2 - 8x - 15$. ✓ (3 marks)



$$a) i) k = 18$$

$$ii) c = \frac{3}{2}$$

13 $f(x) = x^2 + (k-4)x + \left(\frac{1}{2}k + 3\right)$

a Find the discriminant of $f(x)$ in terms of k . (3 marks)

b Given that the equation $f(x) = 0$ has two equal roots, find the possible values of k . (2 marks)

c Show that when $k = 8$, $f(x) > 0$ for all values of x . (3 marks)

a) $\Delta = b^2 - 4ac$
 $= (k-4)^2 - 4\left(\frac{1}{2}k + 3\right)$
 $= k^2 - 8k + 16 - 2k - 12$
 $= k^2 - 10k + 4$

b) $b^2 - 4ac = 0$

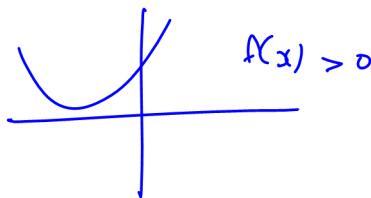
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$k^2 - 10k + 4 = 0$$

$$k = 5 \pm \sqrt{21}$$

c) $k=8$ $f(x) = x^2 + 4x + 7$

$$(x+2)^2 + 3$$



14 The resultant of the forces $F_1 = 6\mathbf{i} + 3p\mathbf{j}$ and $F_2 = 4p\mathbf{i} - 5\mathbf{j}$ acts parallel to $\mathbf{i}$. Find:

a the value of p (2 marks)

b the magnitude of the resultant force. (2 marks)

a) $R = (6+4p)\mathbf{i} + (3p-5)\mathbf{j}$

parallel to $\mathbf{i}$ $\mathbf{j}$ is zero $3p-5 = 0 \Rightarrow p = \frac{5}{3}$

b) $R = \left(6+4 \times \frac{5}{3}\right)\mathbf{i}$

15 $f(x) = \frac{1}{3}\sqrt{x} + \frac{3}{\sqrt{x}}$; $x > 0$

a Find $f'(x)$. (2 marks)

b Find the coordinates of the stationary point on the curve $y = f(x)$. (2 marks)

c By first finding $f''(x)$, determine whether the stationary point found in b is a local minimum or a local maximum. (2 marks)

a) $f(x) = \frac{1}{3}x^{1/2} + 3x^{-1/2}$

$$f'(x) = \frac{1}{6}x^{-1/2} - \frac{3}{2}x^{-3/2}$$

b) $\frac{1}{6}x^{-1/2} - \frac{3}{2}x^{-3/2} = 0$

$$x^{-1/2} \left(\frac{1}{6} - \frac{3}{2}x^{-1} \right) = 0$$

$$x \neq 0 \quad \frac{1}{6} - \frac{3}{2x} = 0 \Rightarrow x = 9$$

$$x=0 \quad f(0)=$$

$$x=9$$

$$f(9)=2$$

$$(9, 2)$$

$$f''(x) > 0 \quad \text{min}$$

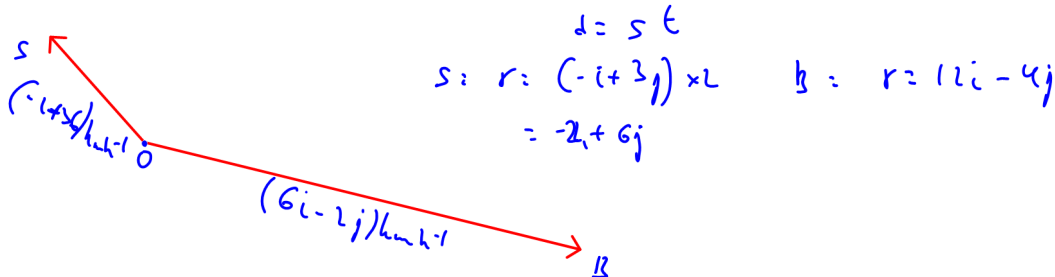
$$f''(x) < 0 \quad \text{max}$$

16 At time $t = 0$, a ship, S , and a boat, B , sail from position O . The ship sails with velocity vector $(-i + 3j)$ km h^{-1} and the boat sails with velocity vector $(6i - 2j)$ km h^{-1} .

a Work out the position vector of the ship and the boat after 2 hours. (2 marks)

b Calculate the distance of the ship from the boat after 2 hours, leaving your answer as a simplified surd. (3 marks)

a)



$$b) d = \sqrt{(12+2)^2 + (-4-6)^2}$$

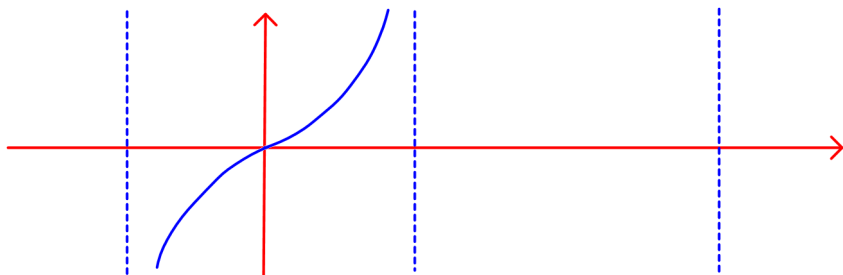
17 The graph of $y = \tan(x + k)$ passes through the point $(60^\circ, 1)$.

a Find one positive and one negative possible value of k . (3 marks)

b Give the equations of two different asymptotes. (3 marks)

$$a) 1 = \tan(60+k) \quad 60+k = 45, 180+45, \\ 60+k = 45, 225 \\ k = -15, 165$$

$$b) y = \tan(x - 15) \quad \text{or} \quad y = \tan(x + 165)$$



$$y = \tan(x - 15) \rightarrow 15$$

$$(15, 0)$$

$$y = \tan(x + 165) \leftarrow 165$$

$$(-165, 0)$$

18 Find the exact value of x such that $\log_{16}(1-3x) = \frac{3}{4}$

(3 marks)

$$1-3x = 16^{3/4} \Rightarrow 1-3x = 8$$
$$x = \frac{1-8}{3}$$

19 The point B lies on the line with equation $3x + 2y = 12$. Given that $|OB| = 10$, find the vector $\overrightarrow{OB}$ in the form $p\mathbf{i} + q\mathbf{j}$, where $p > 0$. (2)

(4 marks)

$$|\overrightarrow{OB}| = p^2 + q^2 = 10^2$$

$$x^2 + y^2 = 100 \quad (1)$$

$$y = -\frac{3}{2}x + 6$$

$$x^2 + \left(-\frac{3}{2}x + 6\right)^2 = 100 \Rightarrow x^2 + \frac{9}{4}x^2 - 18x + 36 = 100$$

Do the rest of questions from q. 20