

Algebraic Proof:

1. Any integer/whole number: n
2. Any even number: $2n$
3. Any odd number: $2n+1$
4. Any three even numbers: $2n, 2m, 2p$
5. Any three odd numbers:
 $2n+1, 2m+1, 2p+1$
6. Any three consecutive even numbers:
 $2n, 2n+2, 2n+4$
7. Any three consecutive odd numbers:
 $2n+1, 2n+3, 2n+5$

* prove that the sum of the squares of any 2 consecutive odd numbers is 2 more than a multiple of 8

$$8(\quad) + 2$$

$$\begin{array}{c} \downarrow \\ 2n+1, 2n+3 \\ \quad \quad \quad \curvearrowright \\ \quad \quad \quad +2 \end{array}$$

$$= (2n+1)^2 + (2n+3)^2$$

$$= (2n+1)(2n+1) + (2n+3)(2n+3)$$

$$= 4n^2 + 4n + 1 + 4n^2 + 12n + 9$$

$$= 8n^2 + 16n + 10$$

$$= 8n^2 + 16n + 8 + 2$$

$$= 8(n^2 + 2n + 1) + 2$$



show that

$$\frac{4}{3+\sqrt{5}} + \sqrt{5} = 3$$

calculator paper, 3 marks

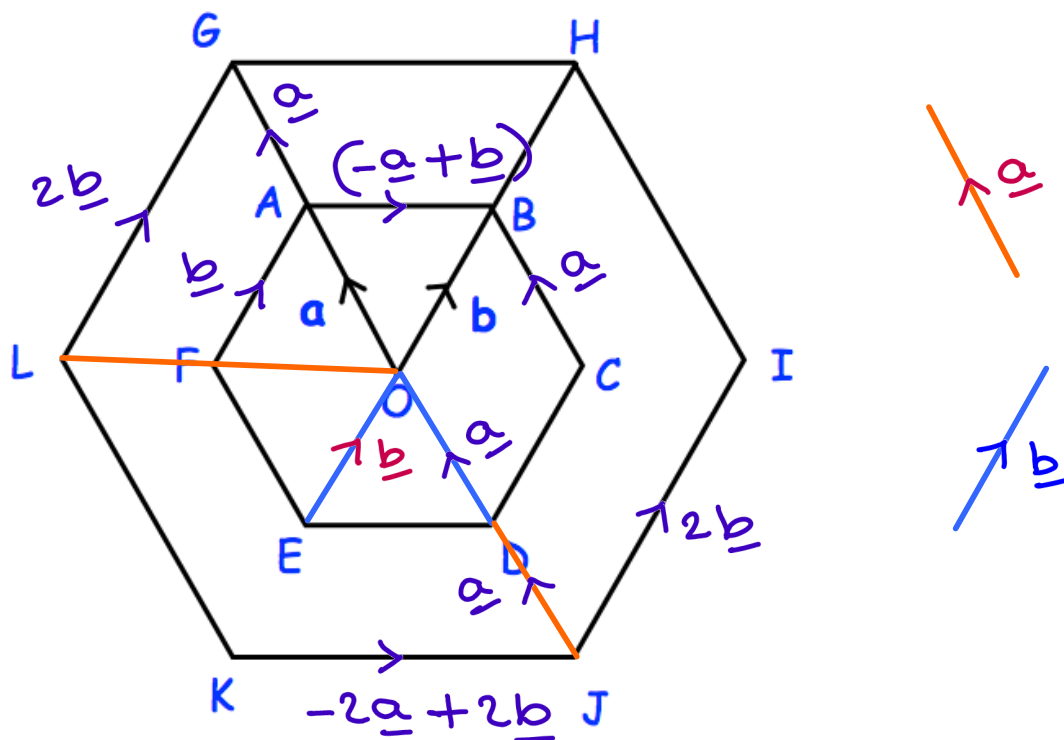
$$\frac{4}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{\cancel{4}(3-\sqrt{5})}{\cancel{4}} + \frac{\sqrt{5}}{1}$$

$$= 3 - \cancel{\sqrt{5}} + \cancel{\sqrt{5}}$$

$$= 3$$

$$(3+\sqrt{5})(3-\sqrt{5}) = 9 - 3\cancel{\sqrt{5}} + 3\cancel{\sqrt{5}} - 5 = 4$$

2.



ABCDEF and GHIJKL are regular hexagons with centre O.
GHIJKL is an enlargement of ABCDEF, with scale factor 2.

$$\overrightarrow{OA} = \underline{a} \quad \text{and} \quad \overrightarrow{OB} = \underline{b}$$

(a) Write the vector $\overrightarrow{AB}$ in terms of $\underline{a}$ and $\underline{b}$.

$$= -\underline{a} + \underline{b}$$

.....
(1)

(b) Write the vector $\overrightarrow{OG}$ in terms of $\underline{a}$ and $\underline{b}$.

$$= 2\underline{a}$$

.....
(1)

(c) Write the vector $\overrightarrow{OE}$ in terms of $\underline{a}$ and $\underline{b}$.

$$= -\underline{b}$$

.....
(1)

(d) Write the vector $\overrightarrow{FC}$ in terms of **a** and **b**.

$$= \underline{b} - \underline{a} + \underline{b} - \underline{a} = 2\underline{b} - 2\underline{a}$$

.....
(1)

(e) Write the vector $\overrightarrow{IK}$ in terms of **a** and **b**.

$$= -2\underline{b} + 2\underline{a} - 2\underline{b} = 2\underline{a} - 4\underline{b}$$

.....
(1)

(f) Write the vector $\overrightarrow{LI}$ in terms of **a** and **b**.

$$2(2\underline{b} - 2\underline{a}) = 4\underline{b} - 4\underline{a}$$

.....
(1)

(g) Write the vector $\overrightarrow{LG}$ in terms of **a** and **b**.

.....
(1)

(h) Write the vector $\overrightarrow{JG}$ in terms of **a** and **b**.

$$= 4\underline{a}$$

.....
(1)

(i) Write the vector $\overrightarrow{DL}$ in terms of **a** and **b**.

$$\underline{a} - 2\underline{b} + 2\underline{a}$$

$$= 3\underline{a} - 2\underline{b}$$

.....
(1)

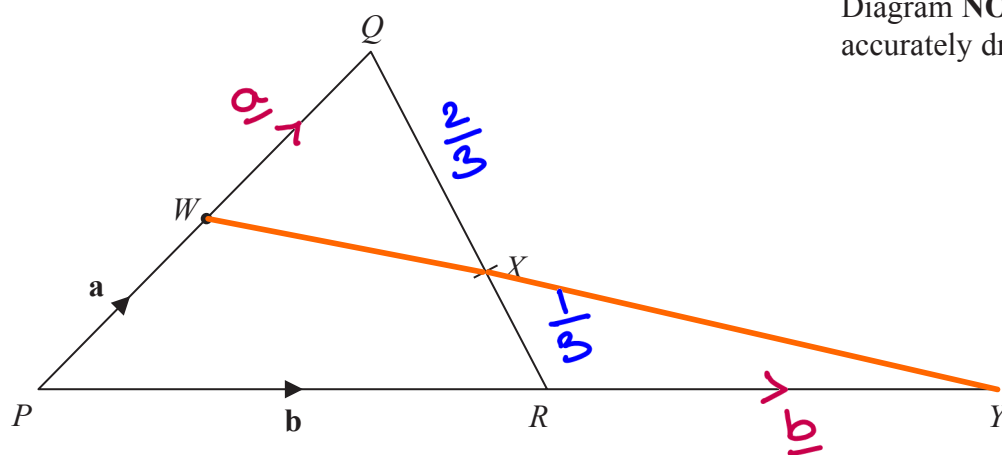


Diagram **NOT**
accurately drawn

PQR is a triangle.

The midpoint of PQ is W .

X is the point on QR such that $QX : XR = 2 : 1$

PRY is a straight line.

$$\overrightarrow{PW} = \mathbf{a} \quad \overrightarrow{PR} = \mathbf{b}$$

(a) Find, in terms of $\mathbf{a}$ and $\mathbf{b}$,

(i) $\overrightarrow{QR} = -2\mathbf{a} + \mathbf{b}$

(ii) $\overrightarrow{QX} = \frac{2}{3}(-2\mathbf{a} + \mathbf{b}) = -\frac{4}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

(iii) $\overrightarrow{WX} = \mathbf{a} - \frac{4}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} = -\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b}$

(3)

R is the midpoint of the straight line PY .

(b) Use a vector method to show that WXY is a straight line.

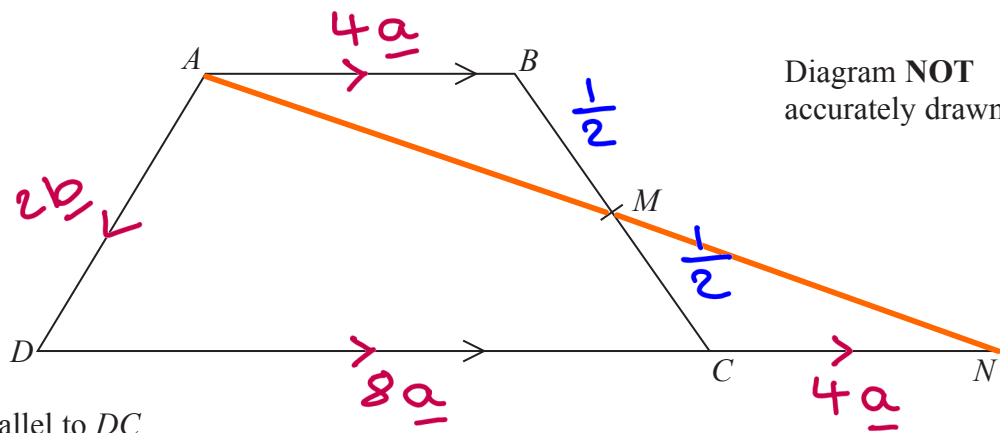
$$\begin{aligned} \overrightarrow{WY} &= -\mathbf{a} + 2\mathbf{b} \\ \overrightarrow{WX} &= -\frac{1}{3}\mathbf{a} + \frac{2}{3}\mathbf{b} \end{aligned}$$

$\times \frac{1}{3}$

(2)

$$\overrightarrow{WX} = \frac{1}{3} \times \overrightarrow{WY} \rightarrow \text{WX and WY are parallel}$$

Both vectors have point W in common



AB is parallel to DC

$DC = 2AB$

M is the midpoint of BC

$\vec{AD} = 2\mathbf{b}$

$\vec{AB} = 4\mathbf{a}$

$$\begin{aligned}\vec{BC} &= -4\mathbf{a} + 2\mathbf{b} + 8\mathbf{a} \\ &= 4\mathbf{a} + 2\mathbf{b}\end{aligned}$$

(a) Find $\vec{BM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Give your answer in its simplest form.

$$\vec{BM} = 2\mathbf{a} + \mathbf{b}$$

(2)

N is the point such that DCN is a straight line and $DC : CN = 2 : 1$

(b) Show that AMN is a straight line.

$$\vec{AM} = 4\mathbf{a} + 2\mathbf{a} + \mathbf{b} = 6\mathbf{a} + \mathbf{b}$$

$$\vec{MN} = 2\mathbf{a} + \mathbf{b} + 4\mathbf{a} = 6\mathbf{a} + \mathbf{b}$$

$$\vec{AM} = \vec{MN} \rightarrow AM \text{ and } MN$$

are parallel (2)

Both vectors have point M in common

$\therefore AMN$ must be a straight line