

CHAPTER 10

10.1 Angles in all four quadrants

- 1

a 1st and 2nd

b 1st and 4th

c 1st and 3rd

d 1st
- 2

a 2nd, negative

b 3rd, positive

c 1st, positive

d 3rd, positive

e 4th, negative

f 4th, positive
- 3

a 0

b -1

c 0

d -1

e -1

f -1

g  $-\frac{1}{\sqrt{2}}$

h  $\frac{1}{\sqrt{2}}$

i undefined
- 4

a  $-\sin \theta$

b  $\cos \theta$

c  $-\tan \theta$

d  $-\sin \theta$

e  $\tan \theta$

f  $-\cos \theta$

g  $\sin \theta$

h  $-\cos \theta$

i  $\tan \theta$

5

a  $-\sin 40^\circ$

b  $-\cos 20^\circ$

c  $\tan 30^\circ$

d  $-\sin 80^\circ$

e  $-\tan 50^\circ$

f  $\cos 80^\circ$

g  $\sin 85^\circ$

h  $-\cos 20^\circ$

i  $\tan 65^\circ$

10.2 Exact values of trigonometric ratios

1

a  $\sin 30^\circ = \frac{1}{2}$

b  $-\sin 45^\circ = -\frac{1}{\sqrt{2}}$

c  $-\sin 45^\circ = -\frac{1}{\sqrt{2}}$

d  $-\sin 60^\circ = -\frac{\sqrt{3}}{2}$

e  $-\cos 30^\circ = -\frac{\sqrt{3}}{2}$

f  $\cos 60^\circ = \frac{1}{2}$

g  $-\cos 45^\circ = -\frac{1}{\sqrt{2}}$

h  $-\cos 60^\circ = -\frac{1}{2}$

i  $\tan 45^\circ = 1$

j  $\tan 30^\circ = \frac{\sqrt{3}}{3}$

k  $\tan 45^\circ = 1$

l  $\tan 60^\circ = \sqrt{3}$

2

$3 + \sqrt{2}$

3

$10^2 = x^2 + (2x)^2 - 2 \times x \times 2x \cos 30^\circ$   
 $\Rightarrow 100 = 5x^2 - 4x^2 \times \frac{\sqrt{3}}{2} \Rightarrow 100 = x^2 (5 - 2\sqrt{3})$   
 $\Rightarrow x^2 = \frac{100}{5 - 2\sqrt{3}} = \frac{100(5 + 2\sqrt{3})}{13}$

10.3 Trigonometric identities

1

a 1

b 6

c 4

2

a  $2 \tan x$

b  $3 \tan x$

c  $\tan^2 x$

3

$$\text{LHS} \equiv \frac{\sin^2 x - \cos^2 x}{1 - \sin^2 x} \equiv \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$$
$$\equiv \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} \equiv \tan^2 x - 1 = \text{RHS}$$

4

a  $-\frac{\sqrt{55}}{8}$

b  $-\frac{3}{\sqrt{55}}$

5

a 6

b  $(2 - 3 \sin \theta)(2 + 3 \sin \theta)$  or  $4 - 9 \sin^2 \theta$

6

$\text{Using } \sin^2 x \equiv 1 - \cos^2 x,$   
 $4 \cos^2 x - 3(1 - \cos^2 x) = 2 \Rightarrow 7 \cos^2 x = 5$

7

$$\text{LHS} \equiv \frac{\cos^2 x - \sin^2 x}{\cos^2 x - \sin x \cos x} \equiv \frac{\cos x + \sin x}{\cos x}$$
$$\equiv \frac{\cos x}{\cos x} + \frac{\sin x}{\cos x} \equiv \text{RHS}$$

8

a  $\frac{2}{3}$

b  $\frac{\sqrt{5}}{3}$

9

a Use  $\sin 45^\circ = \frac{1}{\sqrt{2}}$  and the sine rule,  $\frac{\sin P}{10} = \frac{\sin 45^\circ}{9}$

b  $-\frac{\sqrt{31}}{9}$

10.4 Simple trigonometric equations

1

a  $-36.9^\circ$  (1 d.p.)

b  $x = 216.9^\circ, x = 323.1^\circ$  (1 d.p.)

2

$\theta = 45^\circ, 315^\circ$

3

$x = 48.6^\circ, 131.4^\circ$  (1 d.p.)

4

$\theta = 95.7^\circ, 275.7^\circ$  (1 d.p.)

5

a Line 3: Divided both sides by  $\sin x$  and lost the solution(s) for  $\sin x = 0$ .  
Line 6: Has not identified the negative solution for  $\cos x = \frac{5}{9}$

b  $x = -56.3^\circ, 0.0^\circ, 56.3^\circ$  (1 d.p.)

6

a  $\frac{5}{3}$

b  $59.0^\circ, 239.0^\circ$

7

$\pm 30^\circ, \pm 150^\circ$

8

a Use  $\sin^2 x \equiv 1 - \cos^2 x$ :  $4 \cos^2 x - 3(1 - \cos^2 x) = 1$   
 $\Rightarrow 4 \cos^2 x - 3 + 3 \cos^2 x = 1$

b  $x = 40.9^\circ, 139.1^\circ, 220.9^\circ$  or  $319.1^\circ$

9

a  $x = -63.4^\circ, 14.5^\circ, 116.6^\circ, 165.5^\circ$  (1 d.p.)

b  $x = 80.4^\circ$  or  $279.6^\circ$  (1 d.p.)

10.5 Harder trigonometric equations

1

a  $10^\circ, 110^\circ, 130^\circ, 230^\circ, 250^\circ, 350^\circ$

b  $90^\circ, 270^\circ$

c  $60^\circ, 150^\circ, 240^\circ, 330^\circ$

2

a  $\theta = 138.5^\circ, 341.5^\circ$  (1 d.p.)

b  $\theta = -53.6^\circ, 173.6^\circ$  (1 d.p.)

c  $\theta = 103.7^\circ, 283.7^\circ$  (1 d.p.)

3

a  $\theta = -170^\circ, -110^\circ, -50^\circ, 10^\circ, 70^\circ, 130^\circ$

b  $\theta = 17.0^\circ, 62.0^\circ, 107.0^\circ, 152.0^\circ, 197.0^\circ, 242.0^\circ, 287.0^\circ, 332.0^\circ$  (1 d.p.)

c  $\theta = -98^\circ, 82^\circ$  (nearest degree)

4

a  $x = 39.3^\circ, 80.7^\circ, 219.3^\circ, 260.7^\circ$  (1 d.p.)

b  $x = 31.2^\circ, 75.5^\circ, 151.2^\circ$  (1 d.p.)

c  $x = 27.1^\circ, 72.1^\circ, 117.1^\circ, 162.1^\circ$  (1 d.p.)

d  $x = -156.8^\circ, -66.8^\circ, 23.2^\circ, 113.2^\circ$  (1 d.p.)

5

a  $x = 40^\circ, 80^\circ, 160^\circ, 200^\circ, 280^\circ, 320^\circ$

b  $x = 245^\circ, 335^\circ$

6

a  $x = 15.5^\circ, 105.5^\circ, 195.5^\circ, 285.5^\circ$

b  $x = 20^\circ, 50^\circ, 140^\circ, 170^\circ$

7

a  $23.1^\circ, 96.9^\circ$

b  $152^\circ, 208^\circ$

8

$x = -54.4^\circ, -18.4^\circ, 17.6^\circ, 53.6^\circ$  or  $89.6^\circ$  (1 d.p.)

9

a  $\frac{\sin 2x}{\cos 2x} = 4 \sin 2x \Rightarrow \sin 2x = 4 \sin 2x \cos 2x$   
 $\Rightarrow \sin 2x - 4 \sin 2x \cos 2x = 0$

b  $\sin 2x = 0$  gives  $x = 0^\circ, 90^\circ, 180^\circ$   
 $\cos 2x = \frac{1}{4}$  gives  $x = 37.8^\circ, 142.2^\circ$

10.6 Equations and identities

1

a  $\theta = 19.5^\circ, 161^\circ, 199^\circ, 341^\circ$

b  $\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

c  $\theta = 0^\circ, 11.5^\circ, 168^\circ, 180^\circ, 360^\circ$

d  $\theta = 71.6^\circ, 104^\circ, 252^\circ, 284^\circ$

e  $\theta = 0^\circ, 70.5^\circ, 289^\circ, 360^\circ$

f  $\theta = 21.1^\circ, 38.9^\circ, 81.1^\circ, 98.9^\circ, 141^\circ, 159^\circ, 201^\circ, 219^\circ, 261^\circ, 279^\circ, 321^\circ, 339^\circ$

2

a  $\theta = 0^\circ, 60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ, 360^\circ$

b  $\theta = 0^\circ, 78.7^\circ, 180^\circ, 259^\circ, 360^\circ$

c  $\theta = 270^\circ$

d  $\theta = 45.5^\circ, 150^\circ, 225^\circ, 330^\circ$

3

a  $2 \cos^2 x - (1 - \cos^2 x) - 3 \cos x + 2 = 0$   
 $\Rightarrow 3 \cos^2 x - 4 \cos x + 1 = 0$

b  $x = 0^\circ, 70.5^\circ, 289^\circ, 360^\circ$

4

a  $\theta = 11.5^\circ, 90^\circ, 168.5^\circ$

b  $\theta = 48.2^\circ, 60^\circ, 300^\circ, 311.8^\circ$

c  $\theta = 30^\circ$  or  $150^\circ$

- 5

a

$2 \sin^2 x - 3(1 - \sin^2 x) = 1 \Rightarrow 5 \sin^2 x = 4$
- b

$x = 63.4^\circ, 116.6^\circ, 243.4^\circ, 296.6^\circ$
- 6
- $x = 70.5^\circ, 109.5^\circ$
- 7
- $x = 30^\circ, 150^\circ$
- 8
- a

$$\frac{\sin \theta \sin \theta}{\cos \theta} = 2 \cos \theta + 3 \Rightarrow \sin^2 \theta = 2 \cos^2 \theta + 3 \cos \theta$$

$$\Rightarrow 3 \cos^2 \theta + 3 \cos \theta - 1 = 0$$
- b

$74.7^\circ, 285.3^\circ$
- 9
- $\theta = 430.5^\circ$  or  $438.5^\circ$

Problem solving: Set A

- B

a

$\frac{5}{2}$
- b

$\theta = 48.2^\circ, 228.2^\circ$
- S
- $x = 6.5^\circ, 53.5^\circ, 126.5^\circ, 173.5^\circ$
- G
- $x = -103.2^\circ, -61.8^\circ, 76.8^\circ, 118.2^\circ$

Problem solving: Set B

- B

a

$$\frac{3 \sin^2 x + \cos^2 x}{\cos^2 x} = \frac{3 \sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x}.$$

Use  $\tan^2 x \equiv \frac{\sin^2 x}{\cos^2 x}$  and rearrange equation.
- b

$x = 49.1^\circ, 130.9^\circ, 229.1^\circ, 310.9^\circ$
- S
- a

$4(1 - \sin^2 x) + 9 \sin x - 6 = 0 \Rightarrow 4 \sin^2 x - 9 \sin x + 2 = 0$
- b

$x = 14.5^\circ, 165.5^\circ, 374.5^\circ, 525.5^\circ$
- G
- $x = 0^\circ, 37.3^\circ, 82.7^\circ$