

1 a Find the exact values of x for which $\tan x = \sqrt{3}$ in the interval $-180^\circ \leq x \leq 180^\circ$ **[3 marks]**

- b** Sketch the graph of $y = \tan 2x$ for $-90^\circ \leq x \leq 90^\circ$. State the equations of the asymptotes. [3]

- Calculate the exact value of

- b** The arc length of the sector.



- [2]

- [2]

- [3]

- [4]

- [3]

- [1]

- a** $\operatorname{cosec} x = 2$

- b** $\sec x = -\sqrt{2}$ [3]

- [3]

- [1]

- i** $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$ **ii** $\arcsin\left(\cos\frac{\pi}{3}\right)$

- [2]

- a** State the maximum speed of the particle.

- [3]

- [3]

- [3]**

- i $\sec \theta$

- ii $\cos \theta$

- [5]

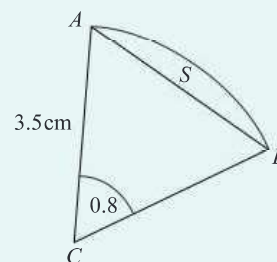
- a** $\cos 75^\circ$

- [3]

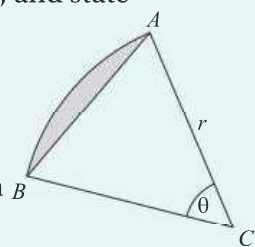
- b** $\cos 15^\circ$

- [2]

- 11** Given that $\sin(A+B) \equiv \sin A \cos B + \sin B \cos A$, show that $\sin 2x \equiv 2 \sin x \cos x$ [2]
- 12** Write $6 \sin \theta + 8 \cos \theta$ in the form $r \sin(\theta + \alpha)$, where $r > 0$ and $0 < \alpha < 90^\circ$ [4]
- 13** For $\theta = 0.05$ radians, state the approximate value of
- a** $\sin \theta$ [1] **b** $\cos \theta$ [1] **c** $\tan \theta$ [1]
- 14 a** Sketch the graphs of these equations for $0 \leq x \leq 2\pi$. Label the x - and y -intercepts with their exact values.
- i** $y = \sin 3x$ **ii** $y = \cos\left(x + \frac{\pi}{3}\right)$ [6]
- b** Solve the equation $\sin 3x = \frac{\sqrt{2}}{2}$ for $0 \leq x \leq \pi$. Show your working and give your answers in terms of π [5]
- 15** A triangle ABC has $AB = 10$ cm, $BC = 13$ cm and $\angle BCA = \frac{\pi}{4}$
- a** Calculate the possible lengths of AC [5]
- b** What is the minimum possible area of the triangle? [3]
- 16** The area of an isosceles triangle is 100 cm^2 . Calculate the perimeter of the triangle, given that one of the angles is $\frac{\pi}{6}$ rad. [7]
- 17** The population (in thousands) of a particular species of insect around a lake t weeks after a predator is released is modelled by
- $$P = 6.5 - 4.1 \sin\left(\frac{\pi t}{2.3}\right)$$
- a** What was the initial population? [2]
- b** State the maximum possible population of the insect. [1]
- c** When does this maximum first occur? Give your answer to the nearest day. [3]
- 18** AB is the arc of a circle with radius 3.5 cm and centre C as shown.
- a** Calculate the
- i** Area of the sector, **ii** Perimeter of the sector. [5]
- The segment S is bounded by the arc and the chord AB
- b** Calculate the
- i** Area of S **ii** Perimeter of S [6]
- 19 a** Sketch these graphs for $0 \leq x \leq 2\pi$. Label the x - and y -intercepts and any asymptotes in terms of π
- i** $y = 2 \operatorname{cosec} x$ **ii** $y = \sec \frac{x}{2}$ [6]
- b** State the range of each of the graphs in part **a**. [3]
- 20** $f(x) = \cot(x - 30^\circ)$
- a** Sketch the graph of $y = f(x)$ for x in the interval $-180^\circ \leq x \leq 180^\circ$. Label the x - and y -intercepts and give the equations of any asymptotes. [4]
- b** Solve the equation $\cot(x - 30^\circ) = 0.2$ for $-180^\circ \leq x \leq 180^\circ$. Show your working. [3]



- 21** Solve these equations for $0 \leq x \leq 2\pi$. Show your working and give your solutions as exact multiples of π
- i $\sec(x+\pi)=2$ ii $\operatorname{cosec}\left(x-\frac{\pi}{8}\right)=\sqrt{2}$ [8]
- 22** $f(x)=\arccos(x-1)$
- a Sketch the graph of $y=f(x)$ for $0 \leq x \leq 2$ [3]
- b State the range of $f(x)$ [2]
- c Work out the inverse of $f(x)$ and state its domain and range. [4]
- 23** Find the exact value of x for which
- a $\arctan(2x-1)=\frac{\pi}{3}$ [2] b $\operatorname{arccot}(x-5)=\frac{2\pi}{3}$ [2]
- 24** a Show that the equation $2\tan^2 x=\sec x-1$ can be written as $2\sec^2 x-\sec x-1=0$ [3]
- b Hence solve the equation $2\tan^2 x=\sec x-1$ for x in the interval $0 \leq x \leq 2\pi$, showing your working. [4]
- 25** a Given that $\sin^2 \theta+\cos^2 \theta \equiv 1$, show that $\operatorname{cosec}^2 \theta-\cot^2 \theta \equiv 1$ [3]
- b Solve the equation $\operatorname{cosec}^2 \theta-3\cot \theta-1=0$ for $0 \leq \theta < 360^\circ$, showing your working. [6]
- 26** a Show that $\sec^4 x-\tan^4 x \equiv \sec^2 x+\tan^2 x$ [2]
- b Find the values of x in the range $-\pi \leq x \leq \pi$ that satisfy $\sec^4 x-\tan^4 x=5+\tan^2 x$. Show your working. [4]
- 27** a By writing $\cos 3x$ as $\cos(2x+x)$, show that $\cos 3x=4\cos^3 x-3\cos x$ [5]
- b Hence solve the equation $8\cos^3 x-6\cos x=\sqrt{3}$ for x in the interval $0 \leq x \leq 2\pi$. Show your working and give your answers as exact multiples of π [5]
- 28** a Use the identity $\cos(A+B) \equiv \cos A \cos B-\sin A \sin B$ to show that $\cos 2x \equiv 2\cos^2 x-1$ [3]
- b Hence solve the equation $\cos 2x+3\cos x+2=0$ for $0 \leq \theta < 360^\circ$, showing your working. [5]
- 29** Prove by counter-example that $\cos(A+B) \neq \cos A+\cos B$ [2]
- 30** $f(x)=8\cos x+4\sin x$
- a Write $f(x)$ in the form $r\cos(x-\alpha)$, where $r>0$ and $0<\alpha<\frac{\pi}{2}$ [4]
- b Hence solve the equation $8\cos x+4\sin x=\sqrt{5}$ for $0<x \leq 2\pi$. Show your working. [4]
- 31** Sketch these graphs for $0 \leq x \leq 360^\circ$
- a $y=2\cos(x+60^\circ)$ [4] b $y=-\sin\left(\frac{x}{2}\right)$ [2]
- 32** $g(x)=\tan\left(\frac{\pi}{3}-x\right)$, for $0 \leq x \leq 2\pi$
- a Sketch the graph of $y=g(x)$, clearly labelling the x - and y -intercepts, and state the equations of the asymptotes. [4]
- b Solve the equation $g(x)=\frac{\sqrt{3}}{3}$. Show your working and give your solutions in terms of π [4]
- 33** The area of sector ABC is 56.7 cm^2 and the length of the arc AB is 12.6 cm . Calculate the area of the shaded segment. [8]



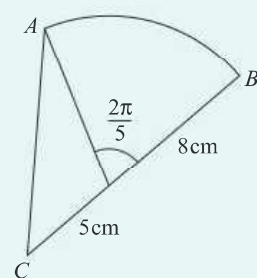
34 The shape ABC is formed from a sector and a triangle as shown in the diagram.

a Find the length of AC

b Calculate

i The area of ABC

ii The perimeter of ABC



[2]

[7]

35 a Sketch the graphs of

i $y = \cot(2x - 60^\circ)$ for $0 \leq x \leq 180^\circ$

ii $y = 1 - \operatorname{cosec}\left(\frac{x}{2}\right)$ for $0 \leq x \leq 360^\circ$

Give the equations of the asymptotes.

[7]

b Solve the equation $\sec(3x + 20) = \sqrt{2}$ for x in the interval $0 \leq x \leq 180^\circ$. Show your working.

[5]

36 a Solve the equation $\cot^2 \theta = 5$ for $0 \leq \theta \leq 2\pi$

[4]

b Find the exact solutions of $\sec^2\left(\theta + \frac{\pi}{6}\right) = 2$ for $0 \leq \theta \leq 2\pi$. Show your working.

[5]

37 Find the coordinates of the points of intersection of the curves $y = \operatorname{cosec} x$ and $y = \sin x$ in the interval $0 \leq x \leq 360^\circ$

[4]

38 a Sketch the graph of $y = |\arcsin x|$ for $-1 \leq x \leq 1$

[2]

b Sketch the graph of $y = 1 + 2\arccos x$ for $-1 \leq x \leq 1$ and label the y -intercept with its exact value.

[3]

39 Show that the curve with Cartesian equation $\frac{x^2}{25} - \frac{y^2}{9} = 1$ has parametric equations $x = 5 \sec \theta$, $y = 3 \tan \theta$

[2]

40 Solve the equation $3 \operatorname{cosec}^2 x - \cot x = 7$ for x in the interval $0 \leq x \leq 360^\circ$. Show your working.

[8]

41 Find all the solutions of $2 \tan^2 2\theta + \sec 2\theta - 4 = 0$ for $0 \leq \theta \leq \pi$. Show your working.

[9]

42 Prove that $\cos \theta \cot \theta - \sin \theta \equiv \operatorname{cosec} \theta - 2 \sin \theta$

[4]

43 Solve the inequality $\cot^2 x > 1 + \operatorname{cosec} x$ for x in the interval $0 \leq x \leq \pi$. Show your working.

[6]

44 a i Prove that $\frac{\cos x}{\sin x} - \frac{\sin x}{1 - \cos x} \equiv -\operatorname{cosec} x$

ii For what values of x is this identity valid?

[5]

b Solve the equation $\frac{\cos x}{\sin x} - \frac{\sin x}{1 - \cos x} = 3$ for $0 \leq x \leq 2\pi$. Show your working.

[4]

45 a Prove that $\cos x \equiv 1 - 2 \sin^2\left(\frac{x}{2}\right)$

[3]

b Find all the solutions of $\cos x + 5 \sin^2\left(\frac{x}{2}\right) = 3$ for $0 < x < 360^\circ$. Show your working.

[6]

46 The height of a tide (in metres) in a harbour t hours after midnight is given approximately by the equation $h = 2.8 + \sqrt{3} \sin\left(\frac{t}{2}\right) - 3 \cos\left(\frac{t}{2}\right)$

A particular boat requires a depth of at least 3.5 m in order to safely leave or enter the harbour.

The owners of the boat wish to depart the harbour in the afternoon.

[8]

47 Solve the equation $\sin 2x + \sin x = 0$ for x in the interval $0 < x < 360^\circ$. Show your working.

[7]

48 Given that $\sin \theta + 2 \cos \theta$ can be written in the form $r \sin(\theta + \alpha)$ where $r > 0$ and $0 < \alpha < 90^\circ$,

a Find the value of r and the value of α

[4]

b Calculate the minimum value of $\frac{1}{(\sin \theta + 2 \cos \theta)^2}$ and the smallest positive value of θ for which this minimum occurs,

[4]

c Find the maximum value of $\frac{1}{3 + \sin \theta + 2 \cos \theta}$ and express it in the form $a + b\sqrt{c}$.

Find also the smallest positive value of θ for which this maximum occurs.

[5]