

06.60

- 4 The first three terms in the binomial expansion of $(1+px)^n$ are $1+12x+24x^2$, where p and n are constants.

Using the series approximation and $x=0.01$,

show that $\sqrt{3} \approx \frac{1403}{810}$

2+7

$$\approx 1 + npx + \frac{n(n-1)}{2!} (px)^2$$

$$= 1 + npx + \frac{n^2-n}{2} p^2 x^2$$

$$n = \frac{12}{p} \quad (n=3/2)$$

$$np = 12$$

$$\frac{n^2-n}{2} p^2 = 24$$

$$n^2 p^2 - n p^2 = 48$$

$$(12)^2 - (np) p = 48$$

$$144 - 12p = 48 \quad (p=8)$$

b) $(1+8x)^{3/2}$

$$x = 0.01 \\ = \frac{1}{100}$$

$$1 + \frac{8}{100} = \frac{27}{25}$$

$$\left(\frac{3\sqrt{3}}{5}\right)^3 = \frac{3^3 \times 3\sqrt{3}}{5^3}$$

$$\begin{aligned} \frac{81}{125} \sqrt{3} &= 1 + 12x + 24x^2 & x &= \frac{1}{100} \\ &= 1 + 12 \times \frac{1}{100} + 24 \times \frac{1}{100}^2 \\ &= \frac{1403}{1250} \end{aligned}$$

$$\sqrt{3} \approx \frac{1403}{\frac{1250}{10}} \times \frac{125}{81} \approx \frac{1403}{810}$$

- 8 a Use the binomial expansion to expand $\sqrt[3]{\frac{1+x}{8-3x}}$ in ascending powers of x , up to and including the term in x^2 , giving each term as a simplified fraction. (8 marks)
- b State the range of values of x for which the expansion is valid. (1 mark)
- c By substituting $x = 0.1$ into your expansion, find an approximation for $\sqrt[3]{7}$, giving your answer to 5 decimal places. (3 marks)

$$a) (1+x)^{1/3} = 1 + \frac{1}{3}x + \frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}x^2$$

$$= 1 + \frac{1}{3}x - \frac{1}{9}x^2$$

$$(8-3x)^{-1/3} = 8^{-1/3} \left(1 - \frac{3}{8}x\right)^{-1/3}$$

$$= \frac{1}{2} \left(1 + \left(-\frac{1}{3}\right)\left(-\frac{3}{8}x\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!} \left(-\frac{3}{8}x\right)^2\right)$$

$$= \frac{1}{2} \left(1 + \frac{1}{8}x + \frac{1}{32}x^2\right)$$

$$= \frac{1}{2} + \frac{1}{16}x + \frac{1}{64}x^2$$

$$\left(1 + \frac{1}{3}x - \frac{1}{9}x^2\right) \left(\frac{1}{2} + \frac{1}{16}x + \frac{1}{64}x^2\right) =$$

$$= \frac{1}{2} + \frac{1}{6}x + \frac{1}{64}x^2 + \frac{1}{6}x + \frac{1}{48}x^2 - \frac{1}{18}x^2 + \dots$$

$$= \frac{1}{2} + \frac{1}{12}x - \frac{11}{576}x^2 + \dots$$

$$b) \sqrt[3]{\frac{1+x}{8-3x}} \quad x = 0.1 \rightarrow \text{Ans}$$

$$\sqrt[3]{\frac{1+0.1}{8-3 \times 0.1}} = \sqrt[3]{\frac{1}{7}} = \frac{1}{\sqrt[3]{7}}$$

$$\frac{1}{\sqrt[3]{7}} = \frac{1}{2} + \frac{11}{48} \text{ Ans} - \frac{11}{576} \text{ Ans}^2$$

$$\approx 0.52273 \quad 5 \text{ d.p.}$$

$$\sqrt[3]{7} = \frac{1}{\text{Ans}} = 1.91305$$

Sequences

Arithmetic: $u_n = a + (n-1)d$
 $S_n = \frac{n}{2} (2a + (n-1)d)$

Geometric: $u_n = a r^{n-1}$

$$S_n = a \frac{1-r^n}{1-r} \quad \text{or} \quad S_n = a \frac{r^n - 1}{r - 1}$$

$$|r| < 1$$

convergent



$$|r| > 1$$

$$S_\infty = a \frac{1-r}{1-r} \quad n \rightarrow \infty \quad r^n \rightarrow 0$$

$$S_\infty = \frac{a}{1-r} \quad |r| < 1 \quad -1 < r < 1$$

4 Evaluate the order of these periodic sequences.

a $u_n = \tan(n \times 60^\circ)$

b $u_{n+1} = 3 - \frac{9}{u_n}, u_1 = 1$

a) $n=1 \quad u_1 = \tan 60 \Rightarrow u_1 = \sqrt{3}$
 $n=2 \quad u_2 = \tan 120 \Rightarrow u_2 = -\sqrt{3}$
 $n=3 \quad u_3 = \tan 180 \Rightarrow u_3 = 0$
 $n=4 \quad u_4 = \tan 240 \Rightarrow u_4 = \sqrt{3}$
 $n=5 \quad u_5 = \tan 300 \Rightarrow u_5 = -\sqrt{3}$

order of 3

$$\sum_{i=1}^{250}$$

$$\frac{250}{3} = 83 + \text{one term}$$

$$(\sqrt{3} - \sqrt{3} + 0) \times 83 + \sqrt{3} = \sqrt{3}$$

b) $u_1 = 1, u_2 = -6, u_3 = \frac{7}{2}, u_4 = 1, u_5 = -6$
order of 3

- $$m_{n+1} = a(m_n - 1)$$

$$m_2 = 4$$

$$m_4 = 0$$

$$u_3 = a(4-1) = 3a$$

$$m_s = a(10-1) = 9a$$

- $$m_{n+1} = a(m_n - 1)$$

$$n=3 \quad m_n = a(m_2-1) \quad \Rightarrow m_n = a(3a-1) = 3a^2 - a = 10$$

$$3a^2 - a - 10 = 0$$

- $a > 2$ or $a = -5/3$

$$m_{n+1} = 2(m_n - 1)$$

$$n=0 \quad m_{12} \quad 2(m_0-1)$$

$$m_2 = 4$$

$$m_4 = 10$$

$$m_2 = 2(m_1 - 1)$$

$$k = 2(m_1 - 1) \Rightarrow$$

$m_1 = 3$

- 3, 4, 6, 10

$$m_{n+1} = 2(m_n - 1)$$

$$u_1 = 3$$

$$m_{n+1} = 2(A_{n5} - 1)$$

3, 4, 6, 12, 18, 36, 66, 130
1 1 1 1 1 1 1

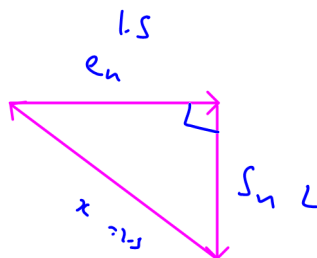
until 8th round

- 7 In preparation to run a race, Paula undertakes weekly training sessions. In the n th session she runs e_n miles due East from her house, turns due South and runs s_n miles and then runs directly back to her house, so that the path she takes in each session is a right-angled triangle.

In the first session she runs 1.5 miles due East and 2 miles due South.

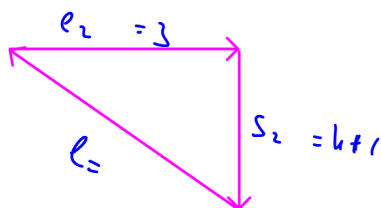
- a Calculate the total distance she runs in session 1

$$d = 1.5 + 2 + 2.5 = 6$$



For $n \geq 1$, it is given that $e_{n+1} = \frac{2}{3}e_n + 2$ and that $s_{n+1} = \frac{1}{2}s_n + k$ where k is a constant. In session 2, Paula runs 12 miles in total.

- b Show that $\sqrt{10+2k+k^2} = 8-k$ and hence evaluate the value of k



$$e_2 + s_2 + l = 12$$

$$e_1 = 1.5 \quad e_2 = \frac{2}{3} \times 1.5 + 2 = 3$$

$$s_1 = 2 \quad s_2 = \frac{1}{2} \times 2 + k = k+1$$

$$e_2 + s_2 + l = 12$$

$$\sqrt{k^2 + 2k + 10} = 8 - k$$

$$l = \sqrt{3^2 + (k+1)^2}$$

$$l = \sqrt{9 + k^2 + 2k + 1}$$

$$l = \sqrt{k^2 + 2k + 10}$$

- 5 Work out the number of terms in each of these arithmetic series.

- a $4 + \dots + 151$ when $S_n = 3875$

$$S_n = \frac{n}{2}(a+l)$$

$$\frac{n}{2}(4+151) = 3875$$

7 Quickline trains has invested in new timetabling software. The number of complaints received from their passengers each month in the following year was modelled using an arithmetic sequence. In the first month, 152 complaints were received. This decreased to 140 complaints in the second month.

$$a = 152 \quad d = -12$$

$$S_{12} = \frac{12}{2} (2 \times 152 + (12-1) \times -12)$$

$$\approx 1032$$

- a Use this model to show that a total of 1032 complaints were received during the year.

The train company claimed that there was roughly a 60% reduction in the number of complaints received between the first six months and the last six months of that year.

$$S_6 = \frac{6}{2} (2 \times 152 + (6-1) \times -12)$$

$$\approx 732$$

- b Use this model to determine whether this claim is accurate.

$$\sum_{n=6}^{12} = 1032 - 732$$

$$= 300$$

$$\frac{500-732}{732} = -59\% \quad \text{around } 60\%$$