

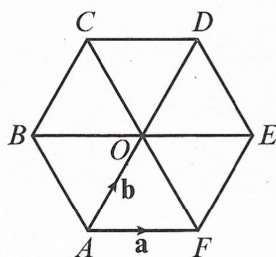
11 Vectors

11.1 Vectors

- 1 $ABCDEF$ is a hexagon. $\overrightarrow{AF} = \mathbf{a}$ and $\overrightarrow{AO} = \mathbf{b}$.

Find in terms of $\mathbf{a}$ and $\mathbf{b}$:

- a $\overrightarrow{AD}$ b $\overrightarrow{EB}$
 c $\overrightarrow{CB}$ d $\overrightarrow{AE}$
 e $\overrightarrow{OF}$ f $\overrightarrow{FC}$
 g $\overrightarrow{CE}$ h $\overrightarrow{BF}$

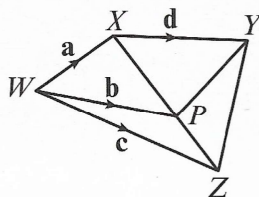


Hint The vector $\overrightarrow{FA} = -\mathbf{a}$ and the vector $\overrightarrow{OA} = -\mathbf{b}$.

- 2 In the diagram, $\overrightarrow{WX} = \mathbf{a}$, $\overrightarrow{WP} = \mathbf{b}$, $\overrightarrow{WZ} = \mathbf{c}$ and $\overrightarrow{XY} = \mathbf{d}$.

Find in terms of $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ and $\mathbf{d}$:

- a $\overrightarrow{XP}$ b $\overrightarrow{PY}$
 c $\overrightarrow{PZ}$ d $\overrightarrow{ZY}$

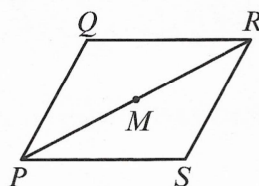


Hint You can use answers to previous parts if it helps. For example, $\overrightarrow{PY} = \overrightarrow{PX} + \overrightarrow{XY}$.

- 3 In the parallelogram $PQRS$, $PQ = \mathbf{a}$ and $QR = \mathbf{b}$. The midpoint of PR is M .

Find in terms of $\mathbf{a}$ and $\mathbf{b}$:

- a $\overrightarrow{PR}$ b $\overrightarrow{PM}$
 c $\overrightarrow{SM}$



Hint The midpoint M is halfway between P and R , so $\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PR}$.

← Section 6.1

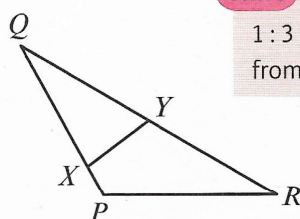
- E** 4 The vector $8\mathbf{a} - 10\mathbf{b}$ is parallel to the vector $p\mathbf{a} + q\mathbf{b}$. State, in simplest terms, the value of $\frac{p}{q}$

(1 mark)

- E/P** 5 In the triangle PQR , $\overrightarrow{PQ} = \mathbf{a}$ and $\overrightarrow{PR} = \mathbf{b}$. X divides PQ in the ratio 1:3 and Y divides QR in the ratio 2:3.

Find $\overrightarrow{YX}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

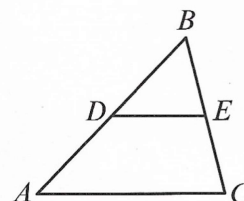
(4 marks)



Hint If X divides PQ in the ratio 1:3 then it is $\frac{1}{4}$ of the way along PQ from P .

- E/P** 6 In the triangle ABC , $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{BC} = \mathbf{b}$. D and E are the midpoints of AB and BC respectively. Use vectors to prove that $\overrightarrow{AC}$ is parallel to $\overrightarrow{DE}$.

(3 marks)



- E/P** 7 $EFGH$ is a trapezium with FG parallel to EH and $\overrightarrow{FG} = k \overrightarrow{EH}$.

M is the midpoint of EF and N is the midpoint of GH .

$\overrightarrow{EH} = \mathbf{a}$, $\overrightarrow{EF} = \mathbf{b}$ and $\overrightarrow{HG} = \mathbf{c}$.

a Prove that $\overrightarrow{MN} = \left(\frac{1+k}{2}\right)\mathbf{a}$. (3 marks)

b Hence, or otherwise, explain why MN is parallel to EH and FG . (1 mark)

11.2 Representing vectors

- 1 $\mathbf{a} = -7\mathbf{i} + 3\mathbf{j}$ and $\mathbf{b} = 2\mathbf{i} - 5\mathbf{j}$.

Find, in terms of $\mathbf{i}$ and $\mathbf{j}$:

a $3\mathbf{a}$ b $\mathbf{a} - 2\mathbf{b}$ c $3\mathbf{b} - \mathbf{a}$

- 2 Given that $\mathbf{a} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$,

find, as a column vector:

a $\frac{1}{2}\mathbf{a}$ b $\mathbf{a} + \mathbf{b} + \mathbf{c}$ c $\mathbf{c} + 2\mathbf{b} - \frac{3}{2}\mathbf{a}$

- 3 Find the resultant of the vectors

$\mathbf{a} = -6\mathbf{i} + 7\mathbf{j}$ and $\mathbf{b} = 3\mathbf{i} - 5\mathbf{j}$.

Hint Add the $\mathbf{i}$ components and add the $\mathbf{j}$ components.

Hint $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$ is another way of writing $-2\mathbf{i} + 4\mathbf{j}$.

Hint The resultant is the vector sum of two or more vectors.

- E** 4 Given that $\overrightarrow{AB} = -8\mathbf{i} + 7\mathbf{j}$ and $\overrightarrow{AC} = 11\mathbf{i} - \mathbf{j}$, find $\overrightarrow{BC}$. (2 marks)

- E** 5 $\mathbf{a} = \begin{pmatrix} 2 \\ p \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} q \\ 11 \end{pmatrix}$
Given that $2\mathbf{b} - \mathbf{c} = \mathbf{a}$, find the value of p and the value of q . (3 marks)

- E/P** 6 $\mathbf{a} = -2\mathbf{i} + 7\mathbf{j}$, $\mathbf{b} = 3\mathbf{i} - 2\mathbf{j}$, $\mathbf{c} = -\mathbf{i} - 6\mathbf{j}$ and $\mathbf{d} = 2\mathbf{i} - 2\mathbf{j}$.
Show that the vector $2\mathbf{a} + 5\mathbf{b}$ is not parallel to the vector $2\mathbf{d} - \mathbf{c}$. (3 marks)

- E/P** 7 $\mathbf{a} = 2p\mathbf{i} + 3q\mathbf{j}$ and $\mathbf{b} = -4q\mathbf{i} + p\mathbf{j}$.
Given that $\mathbf{b} - \mathbf{a} = -10\mathbf{i} + 10\mathbf{j}$, find the value of p and the value of q . (3 marks)

11.3 Magnitude and direction

- 1 Find the exact magnitude of the following vectors.

a $6\mathbf{i} - 8\mathbf{j}$ b $5\mathbf{i} + 4\mathbf{j}$
c $-3\mathbf{i} + 6\mathbf{j}$ d $-8\mathbf{i} - 8\mathbf{j}$

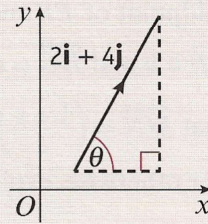
Hint The magnitude of the vector $\mathbf{a} = x\mathbf{i} + y\mathbf{j}$ is $|\mathbf{a}| = \sqrt{x^2 + y^2}$. If you are asked to find the exact magnitude, leave your answer in simplified surd form.

← Section 1.5

- 2 Find the angle that each of these vectors makes with the positive x -axis.

a $2\mathbf{i} + 4\mathbf{j}$ b $-7\mathbf{i} - \mathbf{j}$
 c $6\mathbf{j}$ d $5\mathbf{i} - 5\mathbf{j}$

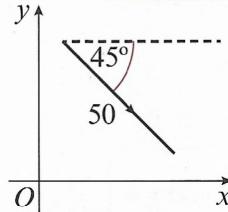
Hint Draw a diagram and calculate the angle θ .



- 3 Find the unit vector in the direction $-5\mathbf{i} + 12\mathbf{j}$.

Hint The unit vector in the direction of $\mathbf{a}$ is $\frac{\mathbf{a}}{|\mathbf{a}|}$

- 4 Write the vector shown in the diagram in the form $p\mathbf{i} + q\mathbf{j}$, where p and q are exact constants to be found.



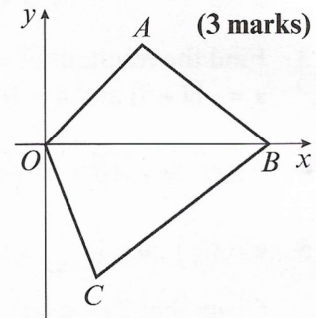
Hint Use trigonometry to find the lengths of the x - and y -components of the vector.

- E/P** 5 The vector $\mathbf{a} = p\mathbf{i} + q\mathbf{j}$, where p and q are positive constants, is such that $|\mathbf{a}| = 11$. Given that $\mathbf{a}$ makes an angle of 20° with the positive y -axis, find the values of p and q . (2 marks)

- E/P** 6 Given that $|2p\mathbf{i} + 3p\mathbf{j}| = 7\sqrt{13}$, find the possible values of p .

- E** 7 Given that $\overrightarrow{OA} = 5\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{AB} = 7\mathbf{i} - 4\mathbf{j}$ and $\overrightarrow{CB} = 10\mathbf{i} + 7\mathbf{j}$, find:

- a $\angle AOB$ (2 marks)
 b $\angle BOC$ (3 marks)
 c $\angle AOC$ (1 mark)



- E/P** 8 Vector $\mathbf{a} = p\mathbf{i} + q\mathbf{j}$ has magnitude 6.5 and makes an angle θ with the positive x -axis where $\cos\theta = -\frac{5}{12}$

Find the possible values of p and q .

(4 marks)

11.4 Position vectors

- 1 The points P and Q have coordinates $(-2, 3)$ and $(7, 0)$ respectively.

Find, in terms of $\mathbf{i}$ and $\mathbf{j}$:

- a the position vector of P
 b the position vector of Q c the vector $\overrightarrow{PQ}$.

Hint $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ where $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ are the position vectors of P and Q respectively.

- 2 The coordinates of the triangle ABC , relative to the origin O , are $A(6, -2)$, $B(-1, -1)$ and $C(1, 3)$. Find, in terms of $\mathbf{i}$ and $\mathbf{j}$:

- a $\overrightarrow{OA}$ b $\overrightarrow{BC}$ c $\overrightarrow{CA}$

Hint $\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$ where $\overrightarrow{OB}$ and $\overrightarrow{OC}$ are the position vectors of B and C respectively.

3 $\overrightarrow{OD} = -6\mathbf{i} + 8\mathbf{j}$ and $\overrightarrow{ED} = -9\mathbf{i} + 3\mathbf{j}$. Find, in surd form:

- a $|\overrightarrow{OD}|$ b $|\overrightarrow{DE}|$ c $|\overrightarrow{OE}|$

Hint Use the given vectors to calculate $\overrightarrow{OE}$.

E 4 $\overrightarrow{OA} = \begin{pmatrix} -7 \\ -6 \end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix} 3 \\ 11 \end{pmatrix}$

Find:

- a the position vector of B (1 mark)
 b the exact value of $|\overrightarrow{OB}|$, giving your answer in surd form. (2 marks)

E/P 5 The point P lies on the line with equation $y = 2x - 3$. Given that $|OP| = \sqrt{53}$, find possible expressions for $\overrightarrow{OP}$ in the form $p\mathbf{i} + q\mathbf{j}$. (4 marks)

11.5 Solving geometric problems

1 Relative to a fixed origin O , the points P and Q have position vectors $\mathbf{p}$ and $\mathbf{q}$ respectively. The point N divides PQ in the ratio 1:3.

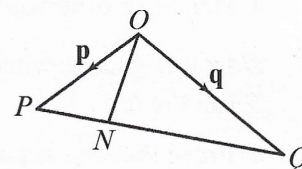
- a Write down $\overrightarrow{PQ}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.
 b Copy and complete this expression by filling the blank space:

$$\overrightarrow{PN} = \frac{\quad}{\quad} \overrightarrow{PQ}$$

- c Copy and complete this expression by filling the blank space:

$$\overrightarrow{ON} = \overrightarrow{OP} + \frac{\quad}{\quad} \overrightarrow{PQ}$$

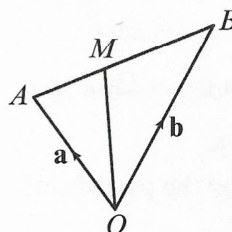
- d Find the position vector of N .



Hint Use the ratio to work out what fraction of $\overrightarrow{PQ}$ is represented by $\overrightarrow{PN}$.

2 Relative to a fixed origin O , the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively. The point M divides AB in the ratio 2:3.

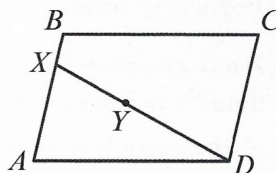
Find the position vector of M in terms of $\mathbf{a}$ and $\mathbf{b}$.



Hint If M divides AB in the ratio 2:3 then it is $\frac{2}{5}$ of the way along AB from A .

3 $ABCD$ is a parallelogram. $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$. X divides AB in the ratio 3:1 and Y divides XD in the ratio 2:3.

Find $\overrightarrow{XY}$.



Hint First find $\overrightarrow{XD}$.

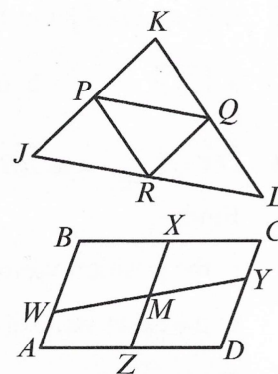
E 4 In triangle PQR , $\overrightarrow{PQ} = 8\mathbf{i} + \mathbf{j}$ and $\overrightarrow{PR} = 2\mathbf{i} - 6\mathbf{j}$.

Find the size of $\angle QPR$ in degrees, correct to one decimal place. (5 marks)

- (E/P) 5** Points P , Q and R are the midpoints of JK , KL and JL respectively.

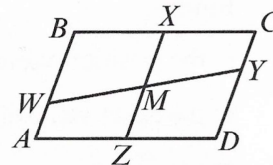
Let $\overrightarrow{JK} = \mathbf{a}$ and $\overrightarrow{JL} = \mathbf{b}$.

- a Prove that $\overrightarrow{PR} = \frac{1}{2}\overrightarrow{KL}$. (3 marks)
 b Explain how you know that $\overrightarrow{PR}$ and $\overrightarrow{KL}$ must be parallel. (1 mark)



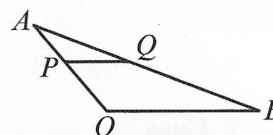
- (E/P) 6** $ABCD$ is a parallelogram. Let $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$. Points X and Z are the midpoints of BC and AD respectively. Points W and Y divide AB and CD in the ratio 1:2 respectively.

- a Find:
 i $\overrightarrow{WY}$ ii $\overrightarrow{XZ}$ (2 marks)
 b Given that $\overrightarrow{WM} = \lambda\overrightarrow{WY}$, where λ is a constant, show that $\overrightarrow{AM} = \frac{1}{3}(1 + \lambda)\mathbf{a} + \lambda\mathbf{b}$. (2 marks)
 c Given also that $\overrightarrow{XM} = \mu\overrightarrow{XZ}$, where μ is a constant, find an expression for $\overrightarrow{AM}$ in terms of $\mathbf{a}$, $\mathbf{b}$, and μ . (2 marks)
 d Hence, or otherwise, prove that M is the midpoint of WY and XZ . (4 marks)



- (E/P) 7** OAB is a triangle where P divides OA in the ratio 1 : k and Q divides BA in the ratio 1 : k .

- a Prove that $\overrightarrow{PQ}$ is parallel to $\overrightarrow{OB}$. (4 marks)
 b Given that $|\overrightarrow{PQ}| = \frac{1}{3}|\overrightarrow{OB}|$, find the value of k . (2 marks)



11.6 Modelling with vectors

- 1 Find the speed of a particle moving with these velocities:

- a $(5\mathbf{i} - 12\mathbf{j}) \text{ m s}^{-1}$ b $(-6\mathbf{i} - 8\mathbf{j}) \text{ km s}^{-1}$
 c $(6\mathbf{i} + \mathbf{j}) \text{ mm s}^{-1}$ d $14\mathbf{j} \text{ cm s}^{-1}$

Hint The speed of a particle travelling with velocity $(x\mathbf{i} + y\mathbf{j}) \text{ m s}^{-1}$ is $\sqrt{x^2 + y^2} \text{ m s}^{-1}$.

- 2 A particle moves with velocity $(-4\mathbf{i} + 3\mathbf{j}) \text{ m s}^{-1}$. Find:

- a the speed of the particle
 b the distance travelled by the particle in 4 seconds
 c the time taken for the particle to travel 27.5 m.

Hint Distance travelled = speed $\times$ time

- 3 At time $t = 0$, a particle P moves with velocity $(-7\mathbf{i} - 24\mathbf{j}) \text{ km s}^{-1}$. The particle accelerates constantly such that, at time $t = 6$, it is moving with velocity $(2\mathbf{i} - 51\mathbf{j}) \text{ km s}^{-1}$.

The acceleration vector of the particle is given by the formula $\mathbf{a} = \frac{\mathbf{v} - \mathbf{u}}{t}$

Find the acceleration of P in terms of $\mathbf{i}$ and $\mathbf{j}$.

- 4 Initially a particle is moving with velocity $(\mathbf{i} - 4\mathbf{j}) \text{ m s}^{-1}$. The particle then accelerates with acceleration $(2\mathbf{i} - 3\mathbf{j}) \text{ m s}^{-2}$ for 4 seconds.

The velocity, $\mathbf{v}$, of the particle after t seconds is given by $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, where $\mathbf{u}$ is the initial velocity and $\mathbf{a}$ is the acceleration. Find:

- a the velocity of the particle after 4 seconds
- b the speed of the particle after 4 seconds.

Hint Substitute the given values of $\mathbf{u}$ and $\mathbf{a}$ into the formula $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ and then simplify your answer.

- E 5** Town A is 3 km west and 6 km north a fixed point O .

Town B is 8 km east and 2 km north of O .

- a Write down the position vector of A relative to O .

(1 mark)

Find:

- b $|\overrightarrow{AB}|$

(3 marks)

- c the bearing of B from A .

(3 marks)

- E 6** The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are given by the vectors $\mathbf{F}_1 = (p\mathbf{i} + q\mathbf{j})$ N and $\mathbf{F}_2 = (3\mathbf{i} - 7\mathbf{j})$ N. The resultant force $\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$ acts in a direction parallel to the vector $(2\mathbf{i} - 3\mathbf{j})$.

- a Find the angle between $\mathbf{R}$ and the positive x -axis.

(2 marks)

- b Show that $3p + 2q = 5$.

(3 marks)

- c Given that $p = 3$, find the magnitude of $\mathbf{R}$.

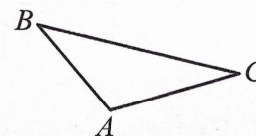
(2 marks)

- E/P 7** A forest fire starts at point A . The fire spreads and covers the area shown by the triangle in the diagram.

The vector $\overrightarrow{AB} = (-3\mathbf{i} + 5\mathbf{j})$ km and the vector $\overrightarrow{AC} = (6\mathbf{i} + \mathbf{j})$ km.

Find the area of forest which is on fire.

(7 marks)



- E 8** A particle P moves under the action of two forces, $\mathbf{F}_1$ and $\mathbf{F}_2$, where $\mathbf{F}_1 = (a\mathbf{i} + b\mathbf{j})$ N and $\mathbf{F}_2 = (-3\mathbf{i} + 7\mathbf{j})$ N. The resultant force acting on the particle is $\mathbf{R} = (8\mathbf{i} + 32\mathbf{j})$ N.

- a Find, to 1 decimal place, the angle between the resultant force and $\mathbf{i}$.

(2 marks)

- b Find the value of a and the value of b .

(3 marks)

A third force $\mathbf{F}_3 = (c\mathbf{i} + 10\mathbf{j})$ N is also applied to P , such that the resultant of all three forces acts parallel to $\mathbf{j}$.

- c Find the value of c .

(3 marks)

Problem solving Set A

Bronze

In triangle ABC , $\overrightarrow{AB} = 5\mathbf{i} + 12\mathbf{j}$ and $\overrightarrow{AC} = 12\mathbf{i} + 5\mathbf{j}$. Find:

- a $\overrightarrow{BC}$

(1 mark)

- b the lengths AB , AC and BC

(3 marks)

- c $\angle BAC$

(2 marks)

Silver

The position vectors of the vertices in triangle ABC are $\vec{OA} = -\mathbf{i} + 4\mathbf{j}$, $\vec{OB} = 8\mathbf{j}$ and $\vec{OC} = 7\mathbf{i} + 9\mathbf{j}$. Find:

- a $\vec{BC}$ (1 mark)
- b $\angle BAC$ (4 marks)
- c the area of triangle ABC . (2 marks)

Gold

In triangle ABC , $\vec{AB} = 8\mathbf{i} - 2\mathbf{j}$ and $\vec{AC} = 2\mathbf{i} - 7\mathbf{j}$. Find the area of triangle ABC . (7 marks)

Problem solving Set B**Bronze**

In triangle OAB , $\vec{OA} = \mathbf{a}$ and $\vec{OB} = \mathbf{b}$.

R divides AB in the ratio $3 : 2$.

$$\vec{OP} = 5\vec{OR}.$$

- a Show that $\vec{OP} = 2\mathbf{a} + 3\mathbf{b}$. (3 marks)

Point Q is such that $\vec{OQ} = -\mathbf{a}$.

- b Prove that the points Q , B and P are collinear. (3 marks)

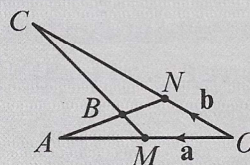
Silver

In the diagram, M is the midpoint of OA and N lies on OC .

The point B lies on MC such that $MB : BC = 1 : 4$.

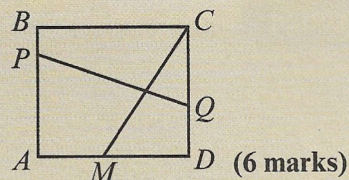
$\vec{OM} = \mathbf{a}$ and $\vec{ON} = \mathbf{b}$. Given that $\vec{AB} = k\vec{AN}$,

- a find the value of k (4 marks)
- b write $\vec{OC}$ as a scalar multiple of $\mathbf{b}$. (1 marks)

**Gold**

$ABCD$ is a square. P divides AB in the ratio $4 : 1$ and Q divides CD in the ratio $3 : 2$. M is the midpoint of AD .

Prove that PQ bisects MC .



Now try this → Exam question bank Q5, Q14, Q16, Q19, Q56, Q80