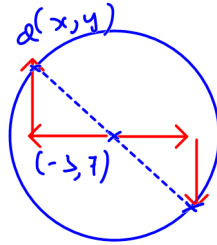


18.09

- 12 The point $P(7, -14)$ lies on the circle with equation $x^2 + y^2 + 6x - 14y = 17$.
The point Q also lies on the circle such that PQ is a diameter.
Find the coordinates of point Q . (4 marks)

$$(x+3)^2 - 3^2 + (y-7)^2 - 7^2 = 14$$

$$(x+3)^2 + (y-7)^2 = 28$$



$$\frac{x+3}{2} = -3 \Rightarrow x = -9$$

$$\frac{y-14}{2} = 7 \Rightarrow y = 28$$

$P(7, -14)$

- 11 A circle C has equation $x^2 + y^2 + 12x + 2y = k$, where k is a constant.

- a Find the coordinates of the centre of C . (2 marks)
b State the range of possible values of k . (2 marks)

$$a) (x+6)^2 - 36 + (y+1)^2 - 1 = k$$

$$(x+6)^2 + (y+1)^2 = k + 37$$

$$b) k + 37 = r^2 \quad r = \sqrt{k + 37}$$

$$k + 37 > 0 \Rightarrow k > -37$$

- 6 a Show that the line $x + y = 11$ meets the circle with equation $x^2 + (y - 3)^2 = 32$ at only one point. (4 marks)
b Find the coordinates of the point of intersection. (1 mark)

$$a) x = 11 - y \quad (11 - y)^2 + (y - 3)^2 = 32$$

$$121 - 22y + y^2 + y^2 - 6y + 9 - 32 = 0$$

$$2y^2 - 28y + 98 = 0 \therefore y^2 - 14y + 49 = 0$$

$$b^2 - 4ac = (-14)^2 - 4 \times 49 = 0 \therefore \text{one solution}$$

$$b) (y - 7)^2 = 0 \quad y = 7 \quad x = 11 - 7 \Rightarrow x = 4$$

$$(4, 7)$$

- 12 The line with equation $y = 2x + 5$ meets the circle with equation $x^2 + kx + y^2 = 4$ at exactly one point. Find two possible values of k .

$$b^2 - 4ac = 0$$

$$x^2 + kx + (2x+5)^2 = 4 \Rightarrow x^2 + kx + 4x^2 + 20x + 25 - 4 = 0$$

$$5x^2 + (k+20)x + 21 = 0$$

$$b^2 - 4ac = 0 \quad \therefore (k+20)^2 - 4 \times 5 \times 21 = 0$$

$$(k+20)^2 = 420 \Rightarrow k+20 = \pm 2\sqrt{105}$$

$$k = -20 \pm 2\sqrt{105}$$

or

$$k = -20 - 2\sqrt{105}$$

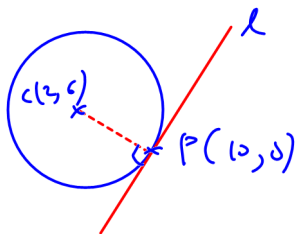
Example 10

The circle C has equation $(x-2)^2 + (y-6)^2 = 100$.

a Verify that the point $P(10, 0)$ lies on C .

b Find an equation of the tangent to C at the point $(10, 0)$, giving your answer in the form $ax + by + c = 0$.

$$a) (10-2)^2 + (0-6)^2 = 64 + 36 = 100 \quad \checkmark$$



$$m_{CP} = \frac{0-6}{10-2} = \frac{-6}{8} = -\frac{3}{4}$$

$$l: m = \frac{4}{3}$$

$$y - 0 = \frac{4}{3}(x - 10)$$

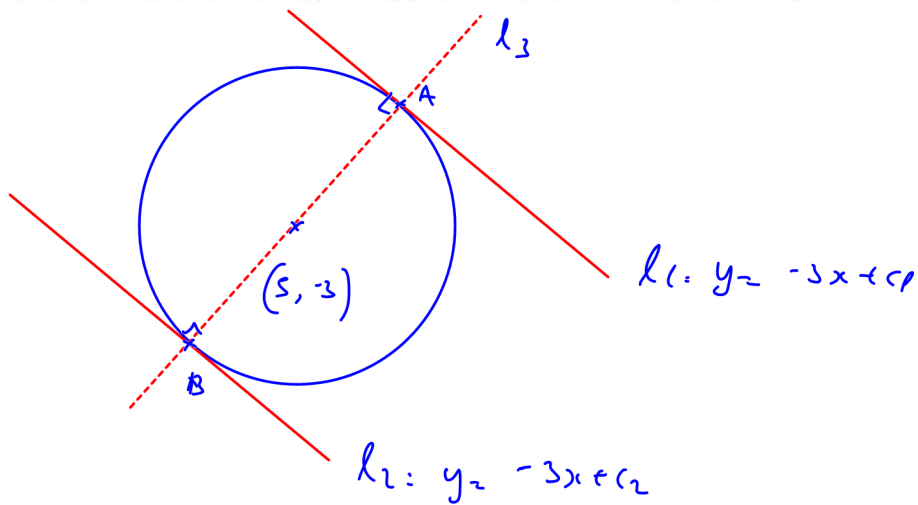
$$3y = 4x - 40 \Rightarrow 4x - 3y - 40 = 0$$

Example 11

A circle C has equation $(x - 5)^2 + (y + 3)^2 = 10$.

The line l is a tangent to the circle and has gradient -3 .

Find two possible equations for l , giving your answers in the form $y = mx + c$.



$$l_3: m = \frac{1}{3} \quad C(5, -3)$$

$$y + 3 = \frac{1}{3}(x - 5) \Rightarrow 3y + 9 = x - 5 \quad x = 3y + 14$$

$$(x - 5)^2 + (y + 3)^2 = 10.$$

$$(3y + 14)^2 + (y + 3)^2 = 10 \Rightarrow 9y^2 + 84y + 81 + y^2 + 6y + 9 = 10 \Rightarrow$$

$$10y^2 + 90y + 80 = 0 \Rightarrow (\div 10) \Rightarrow y^2 + 9y + 8 = 0$$

$$(y + 9)(y + 2) = 0$$

$$y = -9 \Rightarrow x = 3y + 14 \Rightarrow x = -2 \quad (-2, -9)$$

or

$$y = -2 \Rightarrow x = 8 \quad (8, -2)$$

$$l: y = -3x + c \Rightarrow (-2, -9) \Rightarrow -9 = -3 \times 1 + c \Rightarrow c = -6$$

$$y = -3x + 2$$

$$y = -3x + c \Rightarrow (8, -2) \Rightarrow -2 = -3 \times 8 + c \Rightarrow c = 22$$

$$y = -3x + 22$$

Example 12

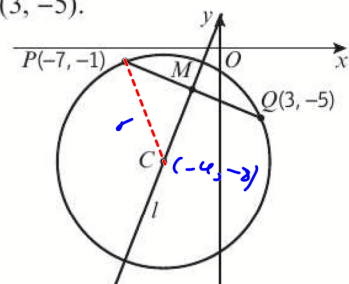
The points P and Q lie on a circle with centre C , as shown in the diagram.
The point P has coordinates $(-7, -1)$ and the point Q has coordinates $(3, -5)$.
 M is the midpoint of the line segment PQ .
The line l passes through the points M and C .

a Find an equation for l .

Given that the y -coordinate of C is -8 ,

b show that the x -coordinate of C is -4

c find an equation of the circle.



$$a) M\left(-\frac{7+3}{2}, \frac{-1-5}{2}\right) \Rightarrow M(-2, -3)$$

$$m_{PQ} = \frac{-5-(-1)}{3-(-7)} = \frac{-4}{10} \Rightarrow m_{PQ} = -\frac{2}{5}$$

$$l: m = \frac{5}{2}, M(-2, -3) \quad y + 3 = \frac{5}{2}(x + 2) \Rightarrow 2y + 6 = 5x + 10$$

$$5x - 2y + 4 = 0$$

$$b) y = -8 \therefore 5x - 2(-8) + 4 = 0 \quad 5x + 20 = 0 \quad x = -4$$

$$(x + 4)^2 + (y + 8)^2 = r^2 \quad P(-7, -1)$$

$$(-7 + 4)^2 + (-1 + 8)^2 = r^2$$

$$9 + 49 = r^2 \quad r^2 = 58$$

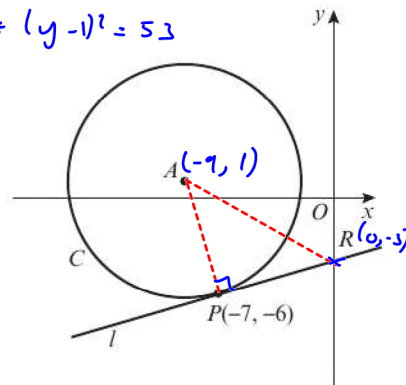
$$(x + 4)^2 + (y + 8)^2 = 58$$

5 The circle C has equation $x^2 + 18x + y^2 - 2y + 29 = 0$. $\Rightarrow (x + 9)^2 + (y - 1)^2 = 53$

a Verify the point $P(-7, -6)$ lies on C . *Plug it in.* (2 marks)

b Find an equation for the tangent to C at the point P , giving your answer in the form $y = mx + b$.

(4 marks)



c Find the coordinates of R , the point of intersection of the tangent and the y -axis. (2 marks)

d Find the area of the triangle APR . (2 marks)

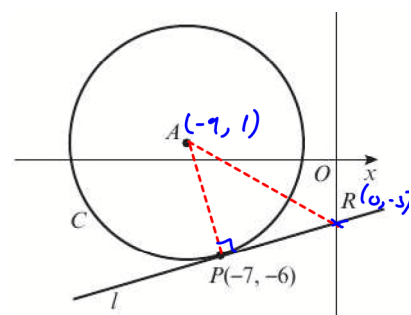
$$b) m_{AP} = \frac{-6-1}{-7-(-9)} = -\frac{7}{2} \Rightarrow l: m = \frac{7}{2} \quad P(-7, -6)$$

$$y + 6 = \frac{7}{2}(x + 7) \Rightarrow 7y + 42 = 3x + 21$$

$$3x - 7y - 21 = 0$$

$$c) \text{ let } x = 0 \therefore -7y - 21 = 0 \quad y = -3 \quad R(0, -3)$$

d) Area = $\frac{1}{2} |AP| \times |PR|$



$$|AP| = \sqrt{(-7+9)^2 + (-6-1)^2} = \sqrt{53}$$

$$|PR| = \sqrt{(0+7)^2 + (-3+6)^2} = \sqrt{58}$$

$$\text{Area} = \frac{1}{2} \sqrt{53} \sqrt{58}$$

11 The circle C has equation $x^2 - 4x + y^2 - 6y = 7$. $\Rightarrow (x-2)^2 + (y-3)^2 = 20$

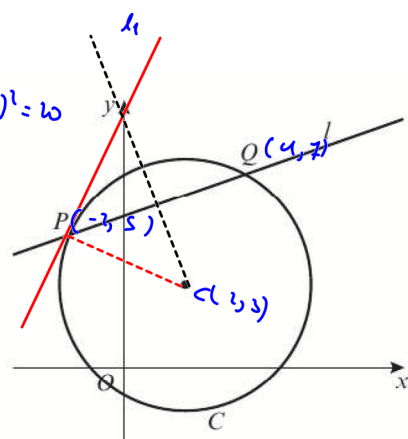
The line l with equation $x - 3y + 17 = 0$ intersects the circle at the points P and Q .

a Find the coordinates of the point P and the point Q . (4 marks)

b Find the equation of the tangent at the point P and the point Q . (4 marks)

c Find the equation of the perpendicular bisector of the chord PQ . (3 marks)

d Show that the two tangents and the perpendicular bisector intersect at a single point and find the coordinates of the point of intersection. (2 marks)



a) $x = 3y - 17 \Rightarrow (3y - 17)^2 - 4(3y - 17) + y^2 - 6y = 7$

$$9y^2 - 102y + 289 - 12y + 68 + y^2 - 6y - 7 = 0$$

$$10y^2 - 110y + 350 = 0 \quad (\div 10) \quad y^2 - 11y + 35 = 0$$

$$(y-7)(y-5) = 0$$

$$y = 7 \Rightarrow x = 3y - 17 \Rightarrow x = 4 \quad (4, 7)$$

$$y = 5 \Rightarrow x = -2 \quad (-2, 5)$$

b) $m_{PC} = \frac{3-5}{2+2} = -\frac{1}{2}$

$l_1: m = 2, P(-2, 5)$

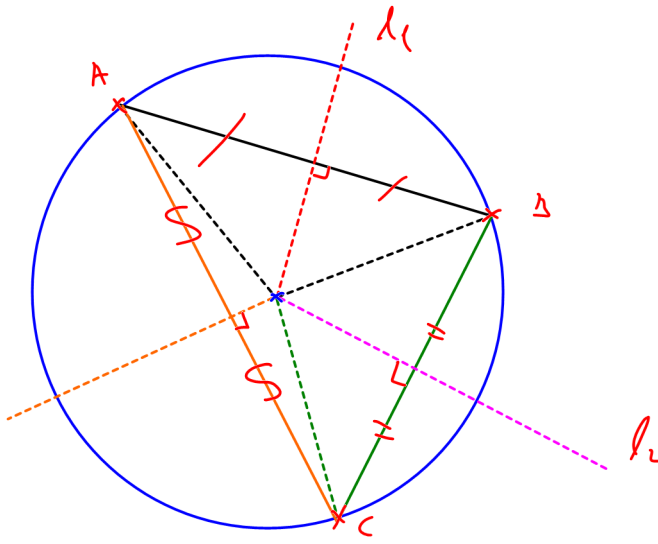
$$y - 5 = 2(x + 2) \quad y = 2x + 9$$

$$M(-2+4, \frac{5+7}{2}) \quad M(1, 6)$$

$$m_{PQ} = \frac{7-5}{4+2} = \frac{2}{6} = \frac{1}{3}$$

$l_2: m = -3 \quad M(1, 6)$

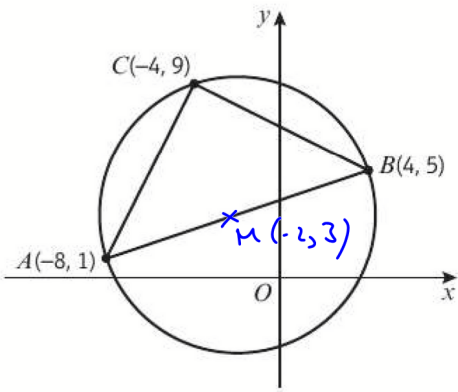
$$y - 6 = -3(x - 1) \Rightarrow y = -3x + 9$$



Example 13

The points $A(-8, 1)$, $B(4, 5)$ and $C(-4, 9)$ lie on the circle, as shown in the diagram.

- Show that AB is a diameter of the circle.
- Find an equation of the circle.



If AB is a diameter then angle ACB is 90°
 $\therefore$ prove that $AC \perp CB$

$$m_{AC} = \frac{9-1}{-4+8} = \frac{8}{4} = 2$$

$$m_{CB} = \frac{5-9}{4+4} = -\frac{4}{8} = -\frac{1}{2}$$

$AC \perp CB \therefore AB$ diameter

$$M\left(-\frac{8+4}{2}, \frac{1+5}{2}\right) \Rightarrow M(-2, 3) \quad B(4, 5)$$

$$(x+2)^2 + (y-3)^2 = r^2$$

$$(4+2)^2 + (5-3)^2 = r^2 \Rightarrow 36 + 4 = r^2 \quad r^2 = 40$$

$$(x+2)^2 + (y-3)^2 = 40$$

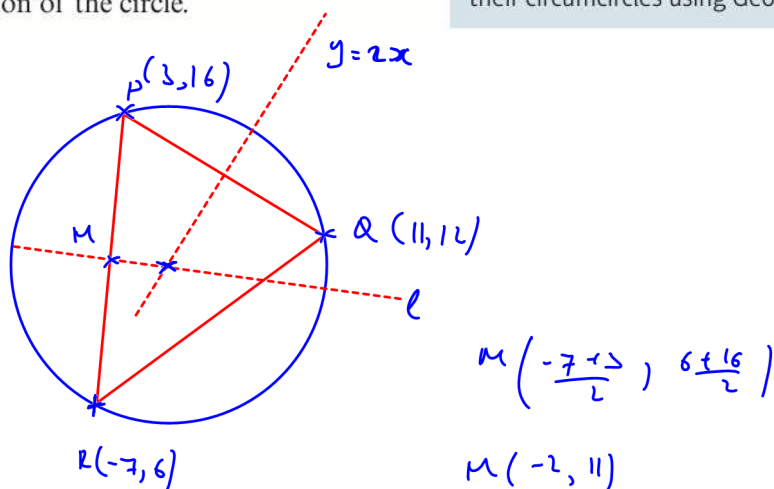
The points $P(3, 16)$, $Q(11, 12)$ and $R(-7, 6)$ lie on the circumference of a circle. The equation of the perpendicular bisector of PQ is $y = 2x$.

- Find the equation of the perpendicular bisector of PR .
- Find the centre of the circle.
- Work out the equation of the circle.

Online Explore triangles and their circumcircles using GeoGebra.



2)



$$m = \frac{16-6}{3-7} = \frac{10}{-4} = -\frac{5}{2}$$

$$l: m = -\frac{5}{2}, M(-2, 11)$$

$$y - 11 = -\frac{5}{2}(x + 2)$$

$$y = -\frac{5}{2}x + 9$$

$y = 2x$ & $y = -\frac{5}{2}x + 9$ they will meet at centre

$$2x = -\frac{5}{2}x + 9 \Rightarrow x = 3 \quad y = 6 \quad (3, 6)$$

$$(x-3)^2 + (y-6)^2 = r^2 \quad P(3, 16)$$

$$(3-3)^2 + (16-6)^2 = r^2$$

$$0 + 100 = r^2$$

$$(x-3)^2 + (y-6)^2 = 100$$

9 A circle has equation $x^2 + 2x + y^2 - 24y - 24 = 0$

a Find the centre and radius of the circle. (3 marks)

b The points $A(-13, 17)$ and $B(11, 7)$ both lie on the circumference of the circle. Show that AB is a diameter of the circle. (3 marks)

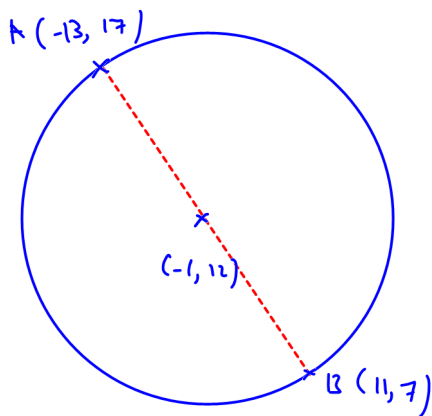
c The point C lies on the negative x -axis and the angle $ACB = 90^\circ$. Find the coordinates of C . (3 marks)

$$a) (x+1)^2 + (y-12)^2 = 169 \quad (-1, 12) \quad r = 13$$

b) If length $AB = 2r = 26$ then AB is a diameter.

$$|AB| = \sqrt{(11+13)^2 + (7-17)^2}$$

$$= 26 \quad \therefore AB \text{ is a diameter}$$



- c The point C lies on the negative x -axis and the angle $ACB = 90^\circ$.
Find the coordinates of C .

(3 marks)

$$(x+1)^2 + (y-12)^2 = 169 \quad y=0$$

$$(x+1)^2 + (0-12)^2 = 169 \quad \Rightarrow (x+1)^2 = 169 - 144$$

$$(x+1)^2 = 25$$

$$x+1 = \pm 5 \quad x = 0$$

$$x+1 = -5 \Rightarrow x = -6$$

$$C(-6, 0)$$