

Example 10

A line is parallel to the line $6x + 3y - 2 = 0$ and it passes through the point $(0, 3)$.
Work out the equation of the line.

$$\rightarrow y = -2x + \frac{1}{3} \quad m = -2$$

$$m = -2 \quad P(x_1, y_1) \quad y - y_1 = m(x - x_1)$$

$$y - 3 = -2(x - 0) \Rightarrow y - 3 = -2x$$

$$y = -2x + 3$$

$$2x + y - 3 = 0$$

Example 11

Work out whether these pairs of lines are parallel, perpendicular or neither:

a $3x - y - 2 = 0$
 $x + 3y - 6 = 0$

b $y = \frac{1}{2}x$
 $2x - y + 4 = 0$

a) $y = 3x - 2$
 $y = -\frac{1}{3}x + 2$

b) $y = 2x + 4$ $y = \frac{1}{2}x$

nothing

$$3 \times \left(-\frac{1}{3}\right) = -1$$

perpendicular

Example 12

A line is perpendicular to the line $2y - x - 8 = 0$ and passes through the point $(5, -7)$.
Find the equation of the line.

$$\downarrow$$

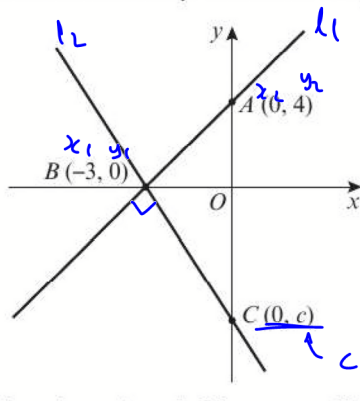
$$y = \frac{1}{2}x + 4 \quad m = \frac{1}{2}$$

perpendicular or $m = -2$ $P(5, -7)$

$$y + 7 = -2(x - 5) \Rightarrow y + 7 = -2x + 10$$

$$y = -2x + 3$$

12 The points A and C lie on the y -axis and the point B lies on the x -axis as shown in the diagram.



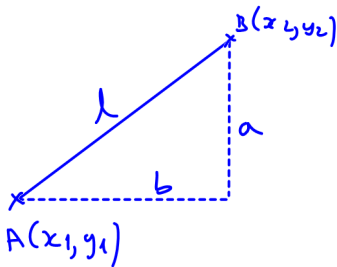
The line through points A and B is perpendicular to the line through points B and C . Find the value of c .

(6 marks)

$$l_1: m = \frac{4-0}{0-(-3)} = \frac{4}{3} \quad l_2: m = -\frac{3}{4} \quad B(-3, 0)$$

$$y - 0 = -\frac{3}{4}(x + 3)$$

$$y = -\frac{3}{4}x - \frac{9}{4}$$



$$a^2 + b^2 = l^2$$

$$b = x_2 - x_1$$

$$a = y_2 - y_1$$

$$l = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example 13

Find the distance between $(2, 3)$ and $(5, 7)$.

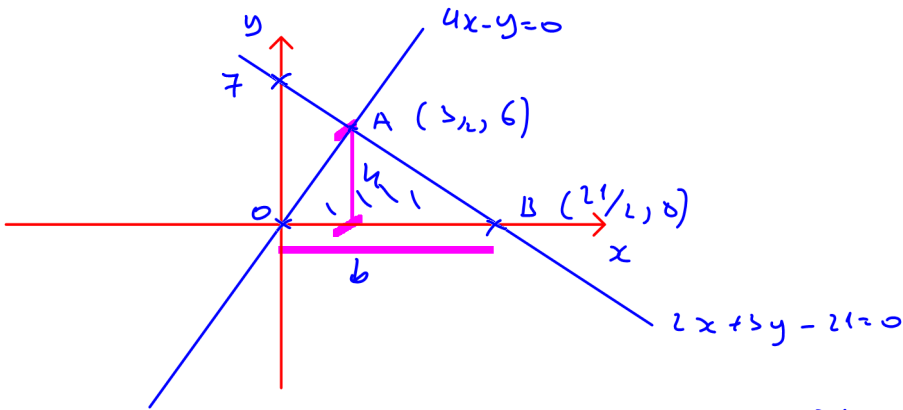
$$\begin{aligned} \text{Distance of } AB &\rightarrow |AB| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5 - 2)^2 + (7 - 3)^2} \\ &= \sqrt{9 + 16} \\ |AB| &= 5 \end{aligned}$$

The straight line l_1 with equation $4x - y = 0$ and the straight line l_2 with equation $2x + 3y - 21 = 0$ intersect at point A .

- Work out the coordinates of A .
- Work out the area of triangle AOB where B is the point where l_2 meets the x -axis.

$$\begin{aligned} a) \quad y &= 4x & 2x + 3 \times 4x - 21 &= 0 \\ & & 14x &= 21 & x &= \frac{21}{14} \Rightarrow x = \frac{3}{2} \end{aligned}$$

$$y = 4 \times \frac{3}{2} \Rightarrow y = 6 \quad \left(\frac{3}{2}, 6 \right)$$



$$B: y = 0 \quad 2x + 21 = 0 \quad x = -21/2$$

$$A = \frac{1}{2}bh \Rightarrow A = \frac{1}{2} \times \frac{21}{2} \times \frac{3}{2} \Rightarrow$$

$$A = \frac{63}{2} \text{ units}^2$$

- 5 The distance between the points $(2, y)$ and $(5, 7)$ is $3\sqrt{10}$. Find two possible values of y .

$$\text{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(5 - 2)^2 + (7 - y)^2} = 3\sqrt{10}$$

$$3^2 + (7 - y)^2 = 9 \times 10 \Rightarrow (7 - y)^2 = 81$$

$$7 - y = \pm \sqrt{81} \Rightarrow 7 - y = \pm 9$$

$$y = 7 \pm 9 \rightarrow y = 7 + 9 = 16$$

$$\rightarrow y = 7 - 9 = -2$$

7 A point P lies on the line with equation $y = 4 - 3x$. The point P is a distance $\sqrt{34}$ from the origin. Find the two possible positions of point P . (5 marks)

$O(0,0)$

$P(a,b)$

$x=a, y=b$

$b = 4 - 3a$

$\sqrt{(a-0)^2 + (b-0)^2} = \sqrt{34}$

$a^2 + b^2 = 34$

$a^2 + (4 - 3a)^2 = 34$

$a^2 + 16 - 24a + 9a^2 = 34 \rightarrow 10a^2 - 24a - 18 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow a = \frac{24 \pm \sqrt{(-24)^2 - 4 \times 10(-18)}}{2 \times 10}$

$a = \frac{24 \pm \sqrt{1296}}{20}$

$a = \frac{24 \pm 36}{20} \rightarrow a = \frac{24+36}{20} = 3$
 $a = \frac{24-36}{20} \rightarrow a = \frac{24-36}{20} = -\frac{3}{5}$

$b = 4 - 3a \rightarrow a = 3 \quad b = 4 - 3 \times 3 = -5$
 $ \rightarrow a = -\frac{3}{5} \quad b = 4 - 3 \times (-\frac{3}{5}) = \frac{29}{5}$

- 12 The straight line l_1 passes through the point $(-4, 14)$ and has gradient $-\frac{1}{4}$
- a Find an equation for l_1 in the form $ax + by + c = 0$, where a, b and c are integers. (3 marks)
 - b Write down the coordinates of A , the point where straight line l_1 crosses the y -axis. (1 mark)
- The straight line l_2 passes through the origin and has gradient 3. The lines l_1 and l_2 intersect at the point B .
- c Calculate the coordinates of B . (2 marks)
 - d Calculate the exact area of $\triangle OAB$. (2 marks)

a) $y - y_1 = m(x - x_1) \Rightarrow y - 14 = -\frac{1}{4}(x + 4) \quad (\times 4)$

$4y - 56 = -x - 4$

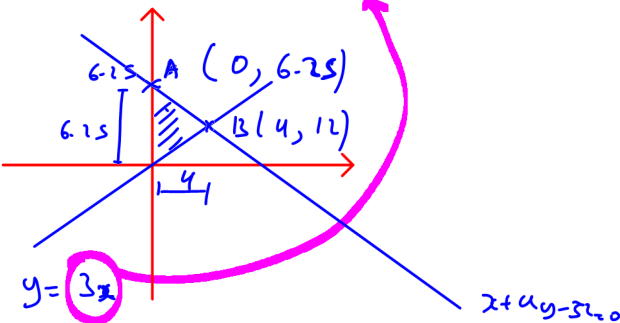
$x + 4y - 52 = 0$

$x + 4(3x) - 52 = 0$

$13x = 52 \quad x = \frac{52}{13} = 4$

$x = 4, y = 3 \times 4 = 12$

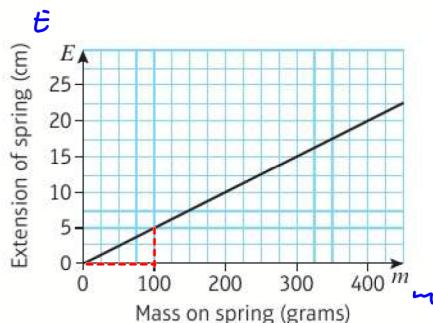
$Area = \frac{1}{2} \times 6.25 \times 4$



Example 15

The graph shows the extension, E , of a spring when different masses, m , are attached to the end of the spring.

- Calculate the gradient, k , of the line.
- Write an equation linking E and m .
- Explain what the value of k represents in this situation.



$$a) k = \frac{E}{m} \Rightarrow k = 0.05 \text{ cm g}^{-1}$$

$$b) y = mx + \frac{0}{c} \quad E = 0.05m$$

c) If mass increases by 1 gram Extension will increase by 0.05 cm

Example 17

In 1991 there were 18 500 people living in Bradley Stoke. Planners projected that the number of people living in Bradley Stoke would increase by 350 each year.

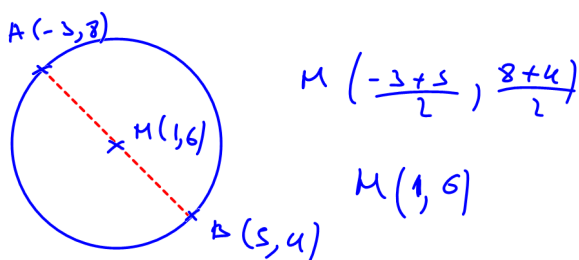
- Write a linear model for the population p of Bradley Stoke t years after 1991.
- Write down one reason why this might not be a realistic model.

$$a) p = 350t + 18,500$$

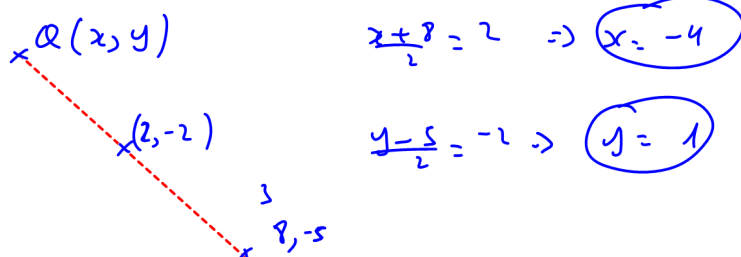
b) You never have a constant rate of increase.

Example 1

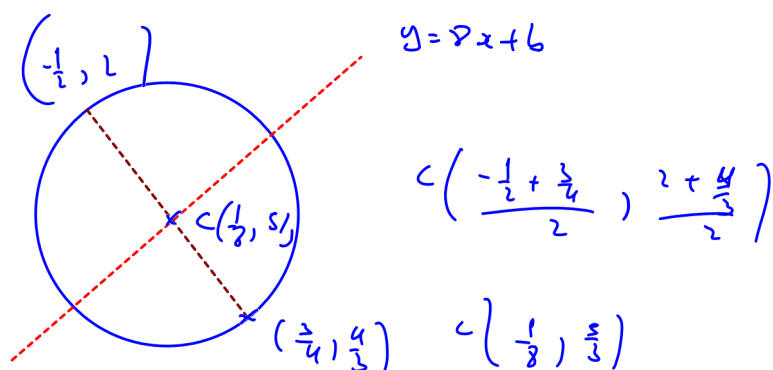
The line segment AB is a diameter of a circle, where A and B are $(-3, 8)$ and $(5, 4)$ respectively. Find the coordinates of the centre of the circle.

**Example 2**

The line segment PQ is a diameter of the circle centre $(2, -2)$. Given that P is $(8, -5)$, find the coordinates of Q .



- 6 The line segment JK is a diameter of a circle, where J and K are $(\frac{3}{4}, \frac{4}{3})$ and $(-\frac{1}{2}, 2)$ respectively. The centre of the circle lies on the line segment with equation $y = 8x + b$. Find the value of b .

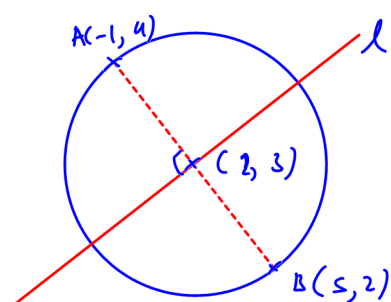


$$x = \frac{1}{8} \quad y = \frac{5}{3} \Rightarrow \frac{5}{3} = 8 \times \frac{1}{8} + b \quad \frac{5}{3} = 1 + b$$

$$b = \frac{5}{3} - 1 \quad b = \frac{5}{3} - \frac{3}{3} \Rightarrow \boxed{b = \frac{2}{3}}$$

Example 3

The line segment AB is a diameter of the circle centre C , where A and B are $(-1, 4)$ and $(5, 2)$ respectively. The line l passes through C and is perpendicular to AB . Find the equation of l .



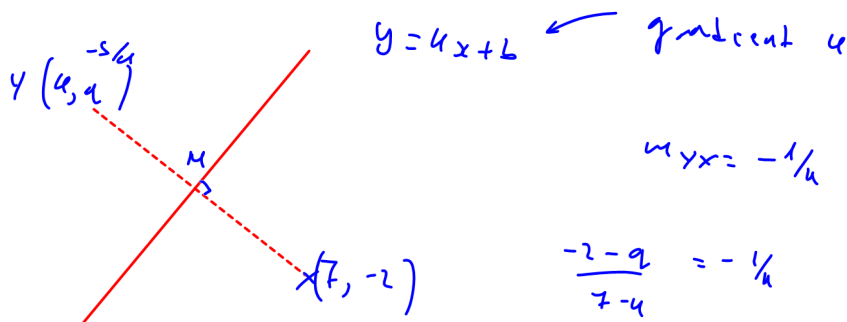
$$m_{AB} = \frac{4-2}{-1-5} = \frac{2}{-6} \Rightarrow m = -\frac{1}{3}$$

$$l: m = 3, C(2, 3)$$

$$y - 3 = 3(x - 2)$$

$$y - 3 = 3x - 6 \Rightarrow \boxed{y = 3x - 3}$$

- 5 Point X has coordinates $(7, -2)$ and point Y has coordinates $(4, q)$. The perpendicular bisector of XY has equation $y = 4x + b$. Find the value of q and the value of b .



$$M = \left(\frac{4+7}{2}, \frac{-2+q}{2} \right)$$

$$= \left(\frac{11}{2}, \frac{-2+q}{2} \right)$$

$$y = 4x + b \quad -\frac{1}{4} = 4 \times \frac{11}{2} + b \quad b = \dots$$