

### Exercise 13.2A Fluency and skills

- 1 a  $u_1 = 8, u_2 = 13, u_3 = 18, u_4 = 23$   
b  $u_1 = -2, u_2 = 1, u_3 = 6, u_4 = 13$   
c  $u_1 = 0, u_2 = 8, u_3 = 36, u_4 = 96$   
d  $u_1 = -\frac{1}{5}, u_2 = -1, u_3 = 1, u_4 = \frac{2}{5}$
- 2 a  $u_2 = 7, u_3 = 19, u_4 = 43$   
b  $u_2 = 3, u_3 = 11, u_4 = 123$   
c  $u_2 = \frac{1}{4}, u_3 = \frac{4}{5}, u_4 = \frac{5}{9}$   
d  $u_2 = 4, u_3 = 1, u_4 = 4$
- 3 a Decreasing      b Increasing  
c Neither          d Neither  
e Decreasing      f Neither
- 4 a 3      b 3      c 4      d 4
- 5 a 77      b  $n > 334$
- 6 a 13      b  $u_4 = 2$
- 7 a  $L = 5$       [ $u_9 = 5.00$  (3 sf)]  
b  $L = -7.5$       [ $u_{16} = -7.50$  (3 sf)]  
c  $L = 2.4$       [ $u_6 = 2.40$  (3 sf)]  
d  $L = -\frac{5}{4}$       [ $u_{13} = -1.25$  (3 sf)]  
e  $L = \frac{2}{3}$       [ $u_{18} = 0.667$  (3 sf)]  
f  $L = 3 + \sqrt{5}$       [ $u_{30} = 5.24$  (3 sf)]  
g  $L = 1 + \sqrt{3}$       [ $u_6 = 2.73$  (3 sf)]  
h  $L = 2$       [ $u_{792} = 2.00$  (3 sf)]

### Exercise 13.2B Reasoning and problem-solving

- 1 a 127      b 166
- 2 a  $u_2 = au_1 + 7$   
 $= a(-1) + 7$   
 $= 7 - a$   
 $u_3 = au_2 + 7$   
 $= a(7 - a) + 7$   
 $= -a^2 + 7a + 7$   
 $-a^2 + 7a + 7 = 19$   
 $a^2 - 7a + 12 = 0$   
b 64 or 83
- 3 a  $a = 2, b = -6$       b 582
- 4 a i  $u_4 = a - \frac{2a}{u_3}$   
 $= a - \frac{2a}{(-2)}$   
 $= 2a$   
So  $u_1 = u_4$  (as the sequence has order 3)  
 $= 2a$   
 $u_2 = a - \frac{2a}{u_1}$   
 $= a - \frac{2a}{2a}$   
 $= a - 1$

- b 300
- 5 a  $p_1 = 56$   
 $p_2 = 0.75(p_1 + 8)$   
 $= 0.75(56 + 8)$   
 $= 48$   
 $p_3 = 0.75(p_2 + 8)$   
 $= 0.75(48 + 8)$   
 $= 42$   
Decrease from 2nd to 3rd observations  $= 48 - 42 = 6$  otters  
% decrease  $= \frac{6}{48} \times 100 = 12.5\%$
- b  $p_n \geq 24$  for some  $n$   
 $p_{n+1} = 0.75(p_n + 8)$   
 $\geq 0.75(24 + 8)$   
 $= 24$   
So at least 24 otters were seen in the  $(n+1)$ th observation
- c Using the same argument as in part b,  
 $p_{n+1} \geq 24$  implies  $p_{n+2} \geq 24$ ,  
which, in turn, implies  $p_{n+3} \geq 24$  and so on  
The number of otters seen from the  $n$ th observation onwards is always at least 24 and hence the population will never fall to zero.
- 6 a  $m_2 = 4$   
 $m_3 = a(m_2 - 1)$   
 $= a(3)$   
 $= 3a$   
 $m_4 = a(m_3 - 1)$   
 $= a(3a - 1)$   
 $m_4 = 10$   
So  $a(3a - 1) = 10$   
leading to  $3a^2 - a - 10 = 0$
- b 3 minutes
- c 7 rounds
- 7 a 6 miles
- b  $e_2 = \frac{2}{3}e_1 + 2$   
 $= \frac{2}{3}(1.5) + 2$   
 $= 3$   
 $s_2 = \frac{1}{2}s_1 + k$   
 $= \frac{1}{2}(2) + k$   
 $= 1 + k$   
Hypotenuse  $= \sqrt{3^2 + (1+k)^2}$   
 $= \sqrt{10 + 2k + k^2}$   
Total distance run  $= 3 + (1+k) + \sqrt{10 + 2k + k^2}$   
So  $4 + k + \sqrt{10 + 2k + k^2} = 12$   
 $\sqrt{10 + 2k + k^2} = 8 - k$ , as required  
 $k = 3$
- c 20 miles

$$\begin{aligned}
 8 \quad a \quad u_2 &= 5p + 6 \\
 &\Rightarrow u_3 = p(5p + 6) + 6 \\
 &= 5p^2 + 6p + 6 \\
 u_3 &= 9.2 \therefore 5p^2 + 6p + 6 = 9.2 \\
 &\Rightarrow 5p^2 + 6p - 3.2 = 0
 \end{aligned}$$

$$b \quad L = 10$$

$$9 \quad a \quad p = 0.8, q = 0.4$$

$$b \quad L = \frac{4}{3}$$

### Exercise 13.3A Fluency and skills

$$\begin{aligned}
 1 \quad a \quad &5, 7, 9, 11 \quad b \quad 9, 6, 3, 0 \\
 c \quad &3, 10, 17, 24 \quad d \quad 4, \frac{13}{2}, 9, \frac{23}{2}
 \end{aligned}$$

$$\begin{aligned}
 2 \quad a \quad u_n &= 4n + 3 \\
 b \quad u_n &= 5 - 3n \\
 c \quad u_n &= \frac{3}{2}n + \frac{5}{2} \text{ or } u_n = \frac{1}{2}(3n + 5)
 \end{aligned}$$

$$d \quad u_n = \frac{2}{3}n + \frac{4}{3} \text{ or } u_n = \frac{2}{3}(n + 2)$$

$$e \quad u_n = \frac{2}{5}n + \frac{1}{5} \text{ or } u_n = \frac{1}{5}(2n + 1)$$

$$f \quad u_n = \sqrt{2}n$$

$$3 \quad a \quad 150 \quad b \quad -139$$

$$c \quad \frac{125}{2} \quad d \quad -102.4$$

$$4 \quad a \quad 585 \quad b \quad -144$$

$$c \quad 2623.5 \quad d \quad 100\sqrt{3}$$

$$5 \quad a \quad 50 \quad b \quad 10$$

$$c \quad 100 \quad d \quad 10$$

$$6 \quad a \quad 480 \quad b \quad 306$$

$$c \quad 119 \quad d \quad 1080$$

### Exercise 13.3B Reasoning and problem-solving

$$1 \quad a \quad d = 5, a = 6 \quad b \quad 61$$

$$2 \quad a \quad \text{Find the first term } a \text{ and the common difference } d$$

$$u_n = a + (n - 1)d$$

$$u_5 = 13 \quad \text{so} \quad a + 4d = 13 \quad \dots [1]$$

$$u_{10} = 7u_2 \quad \text{so} \quad a + 9d = 7(a + d)$$

$$\text{leading to} \quad d = 3a \quad \dots [2]$$

$$\text{Solve [1] and [2] to give } a = 1, d = 3$$

$$u_n = 1 + (n - 1) \times 3$$

$$= 3n - 2$$

$$u_{18} = 3 \times 18 - 2$$

$$= 52$$

$$b \quad 4: u_1 (= 1), u_2 (= 4), u_5 (= 13) \text{ and } u_{18} (= 52)$$

$$3 \quad 475$$

$$4 \quad a \quad u_{16} = 2 \quad b \quad 392$$

$$5 \quad a \quad S_n = \frac{1}{2}n[2 \times 5 + (n - 1) \times 4]$$

$$= \frac{1}{2}n(6 + 4n)$$

$$= n(2n + 3), \text{ as required}$$

$$b \quad 77$$

$$6 \quad a \quad 17 \quad b \quad 790$$

$$7 \quad \text{Let } c_n = \text{number of complaints received in the } n\text{th month.}$$

Let the first term be  $a$  and the common difference  $d$

$$a = 152 (= c_1)$$

$$d = c_2 - c_1$$

$$= 140 - 152$$

$$= -12$$

$$a \quad S_{12} = \frac{1}{2}(12)[2 \times 152 + 11 \times (-12)]$$

$$= 1032$$

The total number of complaints received in the first year was 1032, as required.

$$b \quad S_6 = \frac{1}{2}(6)[2 \times 152 + 5 \times (-12)]$$

$$= 732$$

$$\text{Number of complaints in 2nd half of the year} = S_{12} - S_6$$

$$= 1032 - 732$$

$$= 300$$

Reduction in the number of complaints from 1st to 2nd half of the year =  $732 - 300$

$$= 432$$

$$\% \text{ reduction} = \frac{432}{732} \times 100\%$$

$$= 59.01\%$$

The claim is accurate.

$$c \quad \text{In the following year,}$$

$$n = 13 \text{ and } c_{13} = 152 + 12 \times (-12)$$

= 8 (very low number of complaints, but still possible)

$$n = 14, c_{14} = -4 \text{ which is meaningless}$$

So the model is not suitable for use in the following year.

$$8 \quad a \quad 3300 \quad b \quad £50\,000$$

$$9 \quad a \quad i \quad £142\,500 \quad ii \quad £450\,000$$

$$b \quad 13$$

c The amount Jim earns each year is not likely to increase by the same amount each year forever. Sales made may increase or decrease over time.

$$10 \quad 1^3 + 2^3 + 3^3 + \dots + n^3 \equiv (1 + 2 + 3 + \dots + n)^2; n = 24$$

### Exercise 13.4A Fluency and skills

$$1 \quad a \quad 4, 12, 36, 108 \quad b \quad 3, 12, 48, 192$$

$$c \quad 4, 2, 1, \frac{1}{2} \quad d \quad 2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}$$

$$e \quad \frac{5}{4}, \frac{5}{8}, \frac{5}{16}, \frac{5}{32} \quad f \quad \frac{1}{2}, 1, 2, 4$$

$$2 \quad a \quad u_n = 5 \times 2^{n-1} \quad b \quad u_n = 36 \times \left(\frac{2}{3}\right)^{n-1}$$

$$c \quad u_n = 2 \times (-3)^{n-1} \quad d \quad u_n = (-8) \times \left(-\frac{3}{4}\right)^{n-1}$$

$$e \quad u_n = 7 \times \left(-\frac{7}{2}\right)^{n-1} \quad f \quad u_n = (1) \times \left(\frac{1}{6}\right)^{n-1}$$

$$3 \quad a \quad 6138 \quad b \quad 2059$$

$$c \quad 241 \quad d \quad -5320$$

- 4 a 1594320      b 189  
c 10200      d 32784

### Exercise 13.4B Reasoning and problem-solving

- 1 a  $r = 4, a = \frac{1}{8}$   
b 2048  
c  $u_n = 2^{2n-5}$  so  $p = 2, q = -5$
- 2 a 1820      b 1328600  
3 a 2      b 49146
- 4 a 9310      b  $\frac{3}{16}$  or 0.1875  
5 a 125  
b i 10      ii 1.260
- 6 a 1.02  
b £2653 (nearest £)  
c £56 (nearest £)
- 7 a 0.75  
b  $v_n =$  value of car (in £)  $n$  years after its purchase  
First term  $a = v_1$   
 $= 24000 \times 0.75$   
 $= 18000$   
 $v_n = 18000 \times 0.75^{n-1}$   
so  $v_3 = 18000 \times 0.75^2$   
 $= 10125$   
3 years after purchase the value of the car was £10125, as required.  
c 8 whole years
- 8 a i 1600 miles  
ii 11529 miles  
iii The total distance driven in Tom's lifetime must be less than the sum to infinity of the series  $3125 + 2500 + \dots$   
 $S_\infty = \frac{3125}{1-0.8}$   
 $= 15625$   
Tom will drive less than 15625 miles in his lifetime.  
b The assumption that the distance driven decreases by 20% every year is unrealistic.
- 9 a 6.4 mm  
b  $160 \text{ cm}^3$   
c 439800 km, which is over 55000 km more than the distance from the Earth to the Moon

- 10  $b_n =$  amount due (in £) at end of  $n$ th month  
A geometric model is appropriate since the amount due increased by 5% each month  
So the common ratio  $r = 1.05$   
First term  $a = 450$   
 $b_n = 450 \times 1.05^{n-1}$   
and  $S_n = \frac{450(1.05^n - 1)}{1.05 - 1}$   
 $S_{12} = \frac{450(1.05^{12} - 1)}{1.05 - 1}$   
 $= 7162.706\dots$   
 $\frac{S_{12}}{12} = \frac{7162.706\dots}{12}$   
 $= 596.892\dots$   
Mean monthly bill in 1st year was £600 (nearest £10)  
The bills are increasing in value so median bill = mean of the 6th and 7th terms  
 $b_6 = 450 \times 1.05^5$   
 $= 574.326\dots$   
 $b_7 = 450 \times 1.05^6$   
 $= 603.043\dots$   
Mean of 6th and 7th terms  $= \frac{574.326\dots + 603.043\dots}{2}$   
 $= 588.6848\dots$   
Median bill for 1st year = £590 (nearest £10)  
So the statement used only the mean.
- 11 21 hours