

Exam question bank

This bank of exam-style questions have not been ordered by topic. Read each question carefully to work out which skills and techniques you will need to apply.

- 1 a Find the value of $27^{\frac{5}{3}}$ (2 marks)

- b Simplify fully $\frac{(4x^{\frac{1}{2}})^3}{8x^2}$ (3 marks)

- 2 Simplify $\frac{4\sqrt{3}-2}{7-\sqrt{3}}$, giving your answer in the form $p\sqrt{3}-q$, where p and q are positive rational numbers to be found. (4 marks)

- 3 $f(x) = -2^x + 3$

Sketch the graph of $y = f(x)$, labelling the exact coordinates of any points of intersection with the coordinate axes and the equations of any asymptotes. (4 marks)

- 4 a Simplify $\sqrt{98} - \sqrt{50}$ giving your answer in the form $a\sqrt{2}$, where a is an integer to be found. (2 marks)

- b Hence, or otherwise, simplify

$$\frac{16\sqrt{3}}{\sqrt{98} - \sqrt{50}}$$

giving your answer in the form $b\sqrt{c}$, where b and c are integers and $b \neq 1$. (3 marks)

- 5 Given that $\mathbf{a} = 6\mathbf{i} - 3\mathbf{j}$, find:

- a the unit vector in the same direction of $\mathbf{a}$ (3 marks)

- b the angle that $\mathbf{a}$ makes with the positive x -axis. (2 marks)

- 6 Given that $f(x) = 9\sqrt[3]{x^2} + \frac{4}{x^2} - 3x^3 + 2$, find $\int f(x)dx$. (4 marks)

- 7 Show that $\frac{2\sin^2 x - 1}{\cos^2 x} \equiv \tan^2 x - 1$. (3 marks)

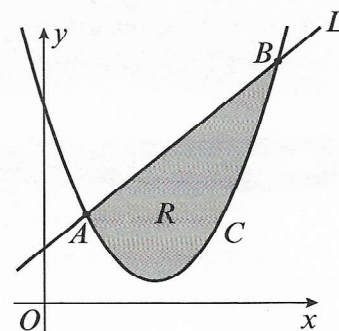
- 8 Find $\int_{\frac{1}{2}}^1 (4-3x)^3 dx$, leaving your answer as a fraction in its simplest terms. (5 marks)

- 9 In the binomial expansion of $(1+3x)^n$, where $n > 0$, the coefficient of x^2 is 6 times the coefficient of x . Work out the value of n . (4 marks)

- 10 The diagram shows part of curve C with equation $y = x^2 - 8x + 18$ and part of the line L with equation $y = 2x + 5$.

- a Using an appropriate algebraic method, find the coordinates of A and B . (4 marks)

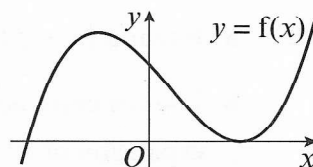
- b Define the shaded region R . (2 marks)



11 $f(x) = 8x^2 - \frac{80}{3}x^{\frac{3}{2}} + 5x; x \geq 0$

Find the x -coordinates of the points on the curve with equation $y = f(x)$ where the gradient is equal to -4 . **(5 marks)**

12 The diagram shows a sketch of part of the curve $y = f(x)$, $x \in \mathbb{R}$, where $f(x) = (2x - 3)^2(x + 2)$.



a i Given that the curve with equation $y = f(x) - k$, $x \in \mathbb{R}$, passes through the origin, find the value of the constant k .

ii Given that the curve with equation $y = f(x + c)$, $x \in \mathbb{R}$, has a local minimum point at the origin, find the value of the constant c . **(3 marks)**

b Show that $f'(x) = 12x^2 - 8x - 15$. **(3 marks)**

13 $f(x) = x^2 + (k - 4)x + \left(\frac{1}{2}k + 3\right)$

a Find the discriminant of $f(x)$ in terms of k . **(3 marks)**

b Given that the equation $f(x) = 0$ has two equal roots, find the possible values of k . **(2 marks)**

c Show that when $k = 8$, $f(x) > 0$ for all values of x . **(3 marks)**

14 The resultant of the forces $\mathbf{F}_1 = 6\mathbf{i} + 3p\mathbf{j}$ and $\mathbf{F}_2 = 4p\mathbf{i} - 5\mathbf{j}$ acts parallel to $\mathbf{i}$. Find:

a the value of p **(2 marks)**

b the magnitude of the resultant force. **(2 marks)**

15 $f(x) = \frac{1}{3}\sqrt{x} + \frac{3}{\sqrt{x}}; x > 0$

a Find $f'(x)$. **(2 marks)**

b Find the coordinates of the stationary point on the curve $y = f(x)$. **(2 marks)**

c By first finding $f''(x)$, determine whether the stationary point found in b is a local minimum or a local maximum. **(2 marks)**

16 At time $t = 0$, a ship, S , and a boat, B , sail from position O . The ship sails with velocity vector $(-\mathbf{i} + 3\mathbf{j}) \text{ km h}^{-1}$ and the boat sails with velocity vector $(6\mathbf{i} - 2\mathbf{j}) \text{ km h}^{-1}$.

a Work out the position vector of the ship and the boat after 2 hours. **(2 marks)**

b Calculate the distance of the ship from the boat after 2 hours, leaving your answer as a simplified surd. **(3 marks)**

17 The graph of $y = \tan(x + k)$ passes through the point $(60^\circ, 1)$.

a Find one positive and one negative possible value of k . **(3 marks)**

b Give the equations of two different asymptotes. **(3 marks)**

18 Find the exact value of x such that $\log_{16}(1 - 3x) = \frac{3}{4}$ (3 marks)

19 The point B lies on the line with equation $3x + 2y = 12$. Given that $|OB| = 10$, find the vector $\overrightarrow{OB}$ in the form $p\mathbf{i} + q\mathbf{j}$, where $p > 0$. (4 marks)

20 a Expand $\left(1 + \frac{2}{x}\right)^2$, simplifying each term. (2 marks)

b Use the binomial expansion to find, in ascending powers of x , the first four terms in the expansion of $\left(1 + \frac{2}{5}x\right)^6$, simplifying each term. (4 marks)

c Hence find the coefficient of x in the expansion of $\left(1 + \frac{2}{x}\right)^2 \left(1 + \frac{2}{5}x\right)^6$. (2 marks)

21 The line l_1 has the equation $2y - 9 = 5x$. The point A with x -coordinate 3 lies on l_1 . The line l_2 is perpendicular to l_1 and passes through the point A .

a Find an equation for l_2 in the form $ax + by + c = 0$. (4 marks)

The lines l_1 and l_2 cross the x -axis at the points B and C respectively.

b Calculate the area of the triangle ABC . (4 marks)

22 The diagram shows a sketch of part of the curve $y = g(x)$ where $g(x) = (x + 1)(2x - 3)^2$

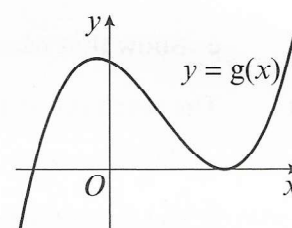
Deduce the values of x for which:

a $g(x) \leq 0$

b $g(2x) = 0$

(1 mark)

(3 marks)



23 The equation $kx^2 + 5kx + 5 = 0$, where k is a constant, has no real roots. Prove that $0 < k < \frac{4}{5}$

(4 marks)

24 a Factorise completely $x^3 + 12x^2 + 36x$

(2 marks)

b Sketch the curve with equation $y = x^3 + 12x^2 + 36x$, showing the coordinates of the points where the curve cuts or touches the x -axis.

(2 marks)

25 Find the exact solutions to the equation $6e^{6x} - 38e^{3x} + 40 = 0$

(5 marks)

26 Given that $\int_9^k \frac{8}{\sqrt{x}} dx = 32$, find the value of k .

(4 marks)

27 $f(x) = ax^3 + bx^2 + cx$

The curve with equation $y = f(x)$ has a gradient of 7 when $x = -1$ and a gradient of 5 when $x = 2$. Additionally, $f''(1) = 2$. Find the values of the constants a , b and c .

(6 marks)

28 Two circles, C_1 and C_2 , have equations $x^2 + (y + 2)^2 = 16$ and $x^2 + (y + 5)^2 = 16$ respectively.

a For each of these circles, state the radius and the coordinates of the centre.

(3 marks)

b Sketch the circles C_1 and C_2 on the same diagram.

(2 marks)

c Find the exact distance between the points of intersection of C_1 and C_2 .

(3 marks)

- 29 a Find the first 4 terms, in ascending powers of p , of the binomial expansion of $(q + p)^{100}$ (3 marks)

Let p represent the probability that any one lightbulb in a batch of 100 is faulty and let q represent the probability it is not faulty. Given that $p = 0.02$,

- b use your answer from part a to work out the probability that no more than 3 lightbulbs in a batch of 100 are faulty. Give your answer correct to 5 decimal places. (5 marks)
- 30 a Prove that when p and q are positive real numbers with $p > q$,
 $\sqrt{p^2 - q^2} > p - q$ (3 marks)

- b Prove by counter-example that this is not true for all values of p and q . (1 mark)

- 31 A parallelogram has sides of lengths 9 cm and 7.5 cm.
 The size of the larger interior angles is θ .
 Given that the area of the parallelogram is 60 cm^2 , find the exact value of $\tan \theta$. (4 marks)

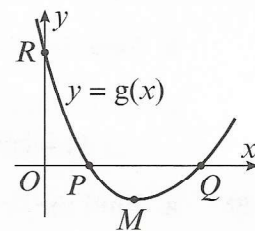
- 32 Find the set of values of x for which:

a $3x - 7 > 3 - 7x$ (2 marks)

b $x^2 - 4x \leq 21$ (4 marks)

c both $3x - 7 > 3 - 7x$ and $x^2 - 4x \leq 21$. (1 mark)

- 33 The diagram shows a sketch of the curve with equation $y = g(x)$.
 The curve has a single turning point, a local minimum, at the point $M(5, -1.5)$.
 The curve crosses the x -axis at two points, $P(3, 0)$ and $Q(8, 0)$.
 The curve crosses the y -axis at a single point $R(0, 9)$.



- a State the coordinates of the turning point on the curve with equation $y = 3g(x)$. (1 mark)
- b State the largest root of the equation $g(x - 2) = 0$. (1 mark)
- c State the range of values of x for which $g'(x) \geq 0$. (1 mark)
- d Given that the equation $g(x) + k = 0$, where k is a constant, has at least one real root, state the range of possible values of k . (1 mark)
- 34 a Use a counter-example to show that the following statement is false.
 ' $5^x \geq 2^x$ for all values of x ' (2 marks)
- b Alison chooses two consecutive odd numbers. She squares them, adds the results, then halves the total. Prove that the result is always an odd number.
 For example, $\frac{1}{2}(9^2 + 11^2) = 101$ (3 marks)