

CHAPTER 6

6.1 Midpoints and perpendicular bisectors

- a $(-1, 1)$ b $(-2, 2)$ c $\left(\frac{3}{5}, -\frac{1}{4}\right)$

d $\left(\frac{\sqrt{3}}{2}, -2\sqrt{5}\right)$ e $(2 + 2\sqrt{2}, 2 - 2\sqrt{2})$
- a $x - 3y - 5 = 0$ b $x - 2y + 12 = 0$

c $4x + 3y + 26 = 0$
- a $(1, 6)$ b $2\sqrt{5}$
- a $a = 9, b = -1$
- a $5y = 16 - 3x$ b $6y = x$ c $\left(\frac{96}{23}, \frac{16}{23}\right)$
- a $x + 3y - 7 = 0$
- a $(1, 2)$ b $4\sqrt{2}$

c $PRQS$ is a square with side length $\sqrt{(4\sqrt{2})^2 + (4\sqrt{2})^2}$
- a 4 b 8

c $2x - 11y + 78 = 0$

6.2 Equation of a circle

- a $(x + 5)^2 + (y - 3)^2 = 64$ b $(x - 7)^2 + (y + 8)^2 = 100$

c $(x + 1)^2 + (y + 4)^2 = 18$
- a $(-2, 9)$, radius 12 b $(5, -2)$, radius $4\sqrt{2}$

c $(-6, 0)$, radius $3\sqrt{5}$
- Centre $(-4, 6)$, radius $\sqrt{67}$
- $(x - 3)^2 + (y - 14)^2 = 52$
- 10 and -4
- a $(3, -5)$ b $5\sqrt{2}$
- a $(x - 1)^2 + (y - 6)^2 = 20$ b $(0, 6 + \sqrt{19}), (0, 6 - \sqrt{19})$
- a $p = -9, q = -2$ b $\sqrt{74}$

c $x^2 + y^2 + 4x - 6y - 61 = 0$
- a $(0, -2), k = 21$ b $5\sqrt{2}$

6.3 Intersections of straight lines and circles

- a $(8, 0)$ and $(-4, 0)$

b $(0, 4 + 4\sqrt{3})$ and $(0, 4 - 4\sqrt{3})$
- $(0, 7)$ and $(6, -5)$
- a Quadratic reduces to $x^2 - 8x + 16 = 0$
 $b^2 - 4ac = 64 - 64 = 0$, so meet at one point.

b $(4, 7)$
- $b^2 - 4ac = -184 < 0$, so line does not meet circle.
- a Circle with centre $(0, 0)$ and radius 10

b $(0, 10)$ and $(6, -8)$
- a Substitute $y = x + k$ into the equation for the circle.

b 5 and -11

c The line is a tangent to the circle.
- a $(-2, 11)$ and $(9, 10)$ b $y = 11x - 28$

c Centre is $(3, 5)$. When $x = 3, y = 11 \times 3 - 28 = 5$ so perpendicular bisector passes through centre of circle.
- $-\frac{40}{9} < k < 0$
- a 25 b 10 and -2

6.4 Use tangent and chord properties

- a Substituting $x = -2$ in the equation gives $y = 10$.

b $5x - 12y + 130 = 0$
- a $(x - 8)^2 + (y - 2)^2 = 80$ b $x + 2y + 8 = 0$
- a $y = 9x + 31$

b Centre is $(-3, 4)$. When $x = -3, y = 9 \times (-3) + 31 = 4$, so perpendicular bisector passes through the centre of circle.
- a $p = -4; k = 1$ or 5

b $3x + y + 9 = 0$
Centre is $(-4, 3); 3(-4) + 3 + 9 = 0$
- a $p = -6, q = -2$ b $\sqrt{13}$

c $x^2 + y^2 - 2x + 8y + 4 = 0$ d $2x - 5y - 51 = 0$
- a Substitute $x = 8$ and $y = 4$ into equation for circle
 $(x + 5)^2 + (y - 9)^2 = 194$

b 51.6

- a $y = 3x - 3$ b 6

c $(x - 3)^2 + (y - 6)^2 = 20$

d $(3 + \sqrt{2}, 6 + 3\sqrt{2})$ and $(3 - \sqrt{2}, 6 - 3\sqrt{2})$
- a $(x - 4)^2 + (y + 1)^2 = 52$ b $y = 19 - 5x$

c 36.1
- a $3x - y - 26 = 0$ b $x - 2y - 7 = 0$ c $(9, 1)$

6.5 Circles and triangles

- a $AB: y = 2x + 3, BC: y = x - 1, AC: 2y = 3x + 2$

b Solve $y = 2x + 3$ and $y = x - 1$ simultaneously to get $x = -4$ and $y = -5$.
These also satisfy $2y = 3x + 2$, so $(-4, -5)$ lies on all three perpendicular bisectors.

c $(x + 4)^2 + (y + 5)^2 = 130$
- a $PR^2 = 392, RQ^2 = 72, PQ^2 = 464, PQ^2 = PR^2 + RQ^2$, so PQ is the hypotenuse of a right-angled triangle and the diameter of the circle.

b $(x + 2)^2 + (y - 5)^2 = 116$
- a $AC^2 = 136, AB^2 = 68, BC^2 = 68, AB = AC$ and $AC^2 = AB^2 + BC^2$ so ABC is an isosceles right-angled triangle.

b $(x - 3)^2 + (y + 2)^2 = 34$
- a $(6, 4)$ b $(x - 6)^2 + (y - 4)^2 = 100$
- a $(5, -3)$ b 50
- a Centre $(2, -4)$, radius $\sqrt{53}$

b E.g. $PQ^2 = (9 + 5)^2 + (-6 + 2)^2 = 212$; diameter $= 2 \times \sqrt{53}$,
So diameter² $= 4 \times 53 = 212$

c $(0, 3)$
- a $(-2, 3)$

b Equation can be written as $(x + 2)^2 + (y - 3)^2 = 36$
So radius is $\sqrt{36} = 6$

c 10.9 (3 s.f.)
- a -15 b $\left(-\frac{5}{2}, -6\right)$
- a 12 b $(x - 4)^2 + (y - 5)^2 = 65$

Problem solving: Set A

- B a $x^2 + (y - 5)^2 = 100$ b $(-6, 13)$ and $\left(\frac{14}{5}, -\frac{23}{5}\right)$
- S Equation of circle: $(x - 5)^2 + (y - 1)^2 = 25$
 $y = \frac{44 - 3x}{4} \Rightarrow x^2 - 10x + 25 + \left(\frac{44 - 3x - 4}{4}\right)^2 = 25$
 $\Rightarrow x^2 - 10x + \left(\frac{1600 - 240x + 9x^2}{16}\right) = 0$
 $\Rightarrow 16x^2 - 160x + 1600 - 240x + 9x^2 = 0$
 $\Rightarrow 25x^2 - 400x + 1600 = 0$
 $\Rightarrow x^2 - 16x + 64 = (x - 8)^2 = 0$
Line meets circle at one point and is a tangent to the circle.
- G $k > 19$ or $k < -63$

Problem solving: Set B

- B a $(2, -3)$ b 5
- c $A(-2, -6)$ and $B(6, -6)$ d $(2, -6)$
- e $\angle APM = \sin^{-1}\left(\frac{4}{5}\right)$ f $AQ = 6.7$
- S a $(4, -5)$ b 12.5
- c $A(-8, -8.5)$ and $B(16, -8.5)$
- d $\angle APB = 2 \times \sin^{-1}\left(\frac{12}{12.5}\right)$ e $AQ = 42.9$
- G a $A(-4.5, -6)$ and $B(10.5, -6)$ b 15.9