

Powers and Roots

Powers are a very useful shorthand:

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7 \quad (\text{'two to the power 7'})$$

That bit is easy to remember. Unfortunately, there are also ten special rules for powers — seven easy ones on this page and three trickier ones on the next page. They're not very exciting, but you need to learn them.

The Seven Easy Rules:



1) When MULTIPLYING, you ADD THE POWERS.

$$\text{e.g. } 3^6 \times 3^4 = 3^{6+4} = 3^{10}, \quad a^2 \times a^7 = a^{2+7} = a^9$$

2) When DIVIDING, you SUBTRACT THE POWERS.

$$\text{e.g. } 5^4 \div 5^2 = 5^{4-2} = 5^2, \quad b^8 \div b^5 = b^{8-5} = b^3$$

3) When RAISING one power to another, you MULTIPLY THEM.

$$\text{e.g. } (3^2)^4 = 3^{2 \times 4} = 3^8, \quad (c^3)^6 = c^{3 \times 6} = c^{18}$$

4) $x^1 = x$, ANYTHING to the POWER 1 is just ITSELF.

$$\text{e.g. } 3^1 = 3, \quad d \times d^3 = d^1 \times d^3 = d^{1+3} = d^4$$

5) $x^0 = 1$, ANYTHING to the POWER 0 is just 1.

$$\text{e.g. } 5^0 = 1, \quad 67^0 = 1, \quad e^0 = 1$$

6) $1^x = 1$, 1 TO ANY POWER is STILL JUST 1.

$$\text{e.g. } 1^{23} = 1, \quad 1^{89} = 1, \quad 1^2 = 1$$

7) FRACTIONS — Apply the power to both TOP and BOTTOM.

$$\text{e.g. } \left(1\frac{3}{5}\right)^3 = \left(\frac{8}{5}\right)^3 = \frac{8^3}{5^3} = \frac{512}{125}, \quad \left(\frac{u}{v}\right)^5 = \frac{u^5}{v^5}$$

Warning: Rules 1 & 2 don't work for things like $2^3 \times 3^7$, only for powers of the same number.

Remember that rules 1 & 2 only work for powers of the same number

If you can add, subtract and multiply, there's nothing on this page you can't do — as long as you learn the rules. Try copying them over and over until you can do it with your eyes closed.

Powers and Roots

The Three Tricky Rules:



8) NEGATIVE Powers — Turn it Upside-Down

People have real difficulty remembering this — whenever you see a negative power you need to immediately think: “Aha, that means turn it the other way up and make the power positive”.

$$\text{e.g. } 7^{-2} = \frac{1}{7^2} = \frac{1}{49} \quad a^{-4} = \frac{1}{a^4} \quad \left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{+2} = \frac{5^2}{3^2} = \frac{25}{9}$$

9) FRACTIONAL POWERS

The power $\frac{1}{2}$ means Square Root,

The power $\frac{1}{3}$ means Cube Root,

The power $\frac{1}{4}$ means Fourth Root etc.

$$\text{e.g. } 25^{\frac{1}{2}} = \sqrt{25} = 5$$

$$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$$

$$81^{\frac{1}{4}} = \sqrt[4]{81} = 3$$

$$z^{\frac{1}{5}} = \sqrt[5]{z}$$

The one to really watch is when you get a negative fraction like $49^{-\frac{1}{2}}$ — people get mixed up and think that the minus is the square root, and forget to turn it upside down as well.

$$\text{e.g. } 49^{-\frac{1}{2}} = \frac{1}{\sqrt{49}} = \frac{1}{7}$$

10) TWO-STAGE FRACTIONAL POWERS

With fractional powers like $64^{\frac{5}{6}}$ always split the fraction into a root and a power, and do them in that order: root first, then power: $(64)^{\frac{1}{6} \times 5} = (64^{\frac{1}{6}})^5 = (2)^5 = 32$.

EXAMPLE:

Simplify $(3a^2b^4c)^3$

Just deal with each bit separately:

$$= (3)^3 \times (a^2)^3 \times (b^4)^3 \times (c)^3$$

$$= 27 \times a^{2 \times 3} \times b^{4 \times 3} \times c^3$$

$$= 27a^6b^{12}c^3$$

You simplify algebraic fractions using the power rules (though you might not realise it).

So if you had to simplify e.g. $\frac{p^3q^6}{p^2q^3}$, you'd just cancel using the power rules to get $p^{3-2}q^{6-3} = pq^3$.

These three rules might be a bit trickier — but they are essential

Because these are things which people often get muddled, examiners love to sneak them into the exam — so scribble down these rules and learn them. Then in the exam you'll have the last laugh.

APPLYING LIMITS OF ACCURACY

ADDITION:

$$\text{LOWER BOUND} = \text{LOWER BOUND} + \text{LOWER BOUND}$$

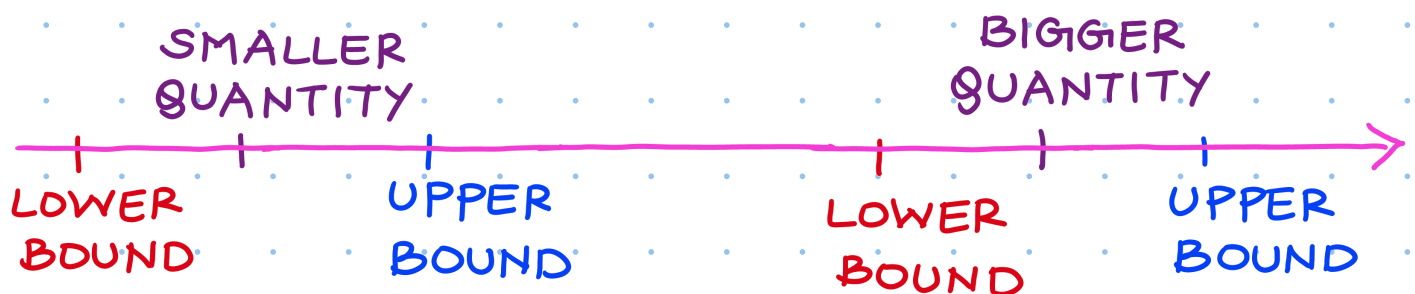
$$\text{UPPER BOUND} = \text{UPPER BOUND} + \text{UPPER BOUND}$$

MULTIPLICATION

$$\text{LOWER BOUND} = \text{LOWER BOUND} \times \text{LOWER BOUND}$$

$$\text{UPPER BOUND} = \text{UPPER BOUND} \times \text{UPPER BOUND}$$

SUBTRACTION: (USE A NUMBER LINE)



$$\text{LOWER BOUND} = \text{LOWER BOUND (BIGGER)} - \text{UPPER BOUND (SMALLER)}$$

$$\text{UPPER BOUND} = \text{UPPER BOUND (BIGGER)} - \text{LOWER BOUND (SMALLER)}$$

DIVISION :

$$\text{LOWER BOUND} = \frac{\text{LOWER BOUND}}{\text{UPPER BOUND}}$$

$$\text{UPPER BOUND} = \frac{\text{UPPER BOUND}}{\text{LOWER BOUND}}$$

$$\text{UPPER BOUND OF SPEED} = \frac{\text{COVER A GREATER DISTANCE (UPPER BOUND)}}{\text{IN A SHORTER TIME (LOWER BOUND)}}$$

$$\text{LOWER BOUND OF SPEED} = \frac{\text{COVER A LESSER DISTANCE (LOWER BOUND)}}{\text{IN A LONGER TIME (UPPER BOUND)}}$$

QUADRATIC FORMULA

General form of a quadratic equation

$$ax^2 + bx + c = 0$$

The quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Completing the square ('a' must be 1)

$$\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

QUADRATIC SEQUENCES

n^{th} term of a quadratic sequence will be in the form

$$an^2 + bn + c$$

To find the coefficients a, b and c

$$a + b + c = \text{first term}$$

$$3a + b = \text{second term} - \text{first term}$$

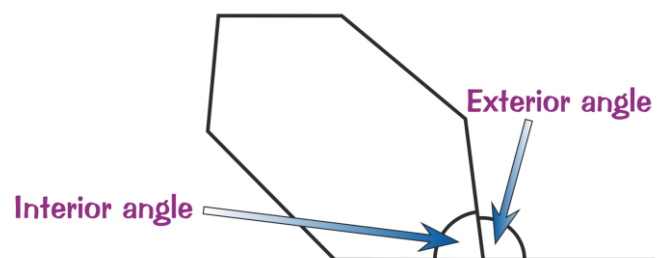
$$2a = \text{second difference}$$

POLYGONS

For any polygon (regular or irregular)

$$\text{Sum of exterior angles} = 360^\circ$$

$$\text{Sum of interior angles} = (n - 2) \times 180^\circ$$



COMPOUND MEASURES

Speed

$$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$



$$\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$$

$$\text{TIME} = \frac{\text{DISTANCE}}{\text{SPEED}}$$

$$\text{DISTANCE} = \text{SPEED} \times \text{TIME}$$

Density

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$



$$\text{DENSITY} = \frac{\text{MASS}}{\text{VOLUME}}$$

$$\text{VOLUME} = \frac{\text{MASS}}{\text{DENSITY}}$$

$$\text{MASS} = \text{DENSITY} \times \text{VOLUME}$$

Pressure

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$



$$\text{PRESSURE} = \frac{\text{FORCE}}{\text{AREA}}$$

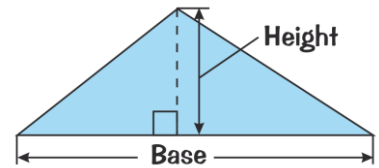
$$\text{AREA} = \frac{\text{FORCE}}{\text{PRESSURE}}$$

$$\text{FORCE} = \text{PRESSURE} \times \text{AREA}$$

AREAS OF TRIANGLES AND QUADRILATERALS

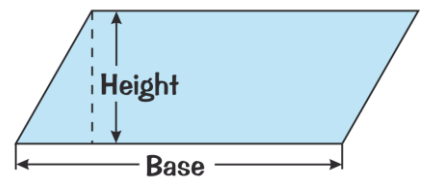
Triangle

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{vertical height}$$



Parallelogram

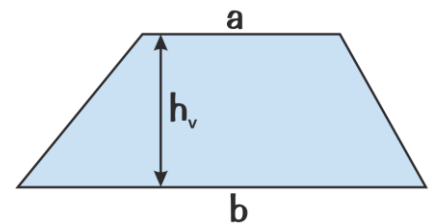
$$\text{Area of parallelogram} = \text{base} \times \text{vertical height}$$



Trapezium

$$\text{Area of trapezium} = \text{average of parallel sides} \times \text{vertical height}$$

$$\text{Area of trapezium} = \frac{1}{2}(a + b)h_v$$



CIRCLE

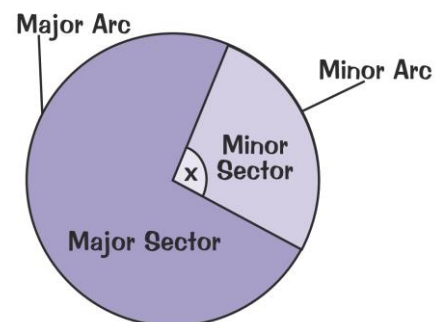
$$\text{Area of a circle} = \pi \times (\text{radius})^2$$

$$\text{Circumference} = 2 \times \pi \times \text{radius}$$

Sector

$$\text{Area of sector} = \frac{x}{360} \times \pi r^2$$

$$\text{Circumference of sector} = \frac{x}{360} \times 2\pi r$$



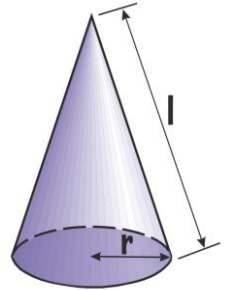
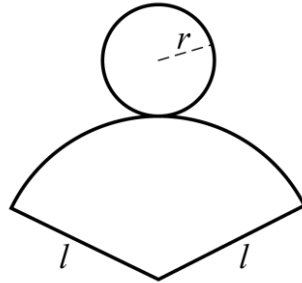
SURFACE AREA

Sphere

$$\text{Surface area of a sphere} = 4\pi r^2$$

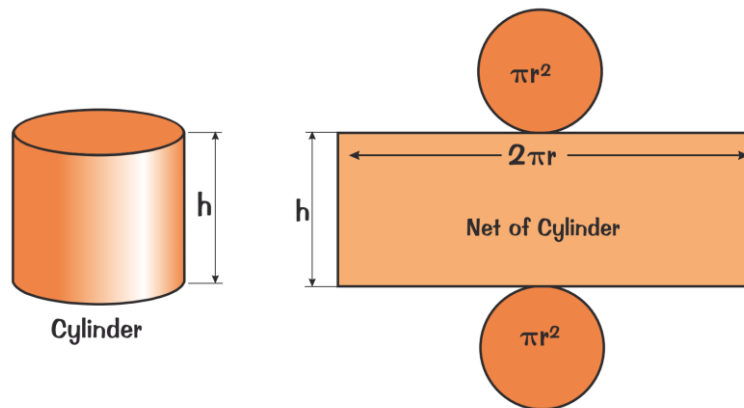
Cone

$$\text{Surface area of a cone} = \pi r l + \pi r^2$$



Cylinder

$$\text{Surface area of a cylinder} = 2\pi r h + 2\pi r^2$$



VOLUME

Prism

$$\text{Volume of a prism} = \text{Cross sectional area} \times \text{length}$$

Sphere

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

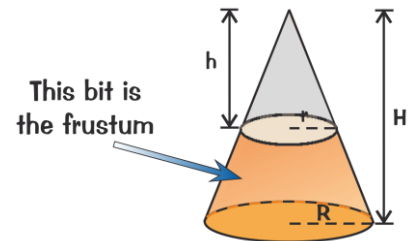
Volumes of pyramids and cones

$$\text{Volume of a pyramid} = \frac{1}{3} \times \text{base area} \times \text{vertical height}$$

Volumes of Frustums

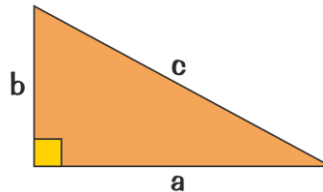
Volume of frustum

$$= V_{\text{original pyramid}} - V_{\text{removed pyramid}}$$



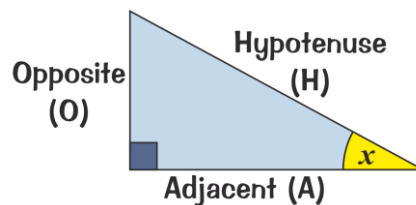
PYTHAGORAS THEOREM

$$a^2 + b^2 = c^2$$



TRIGONOMETRY

$$\sin x = \frac{\text{Opposite}}{\text{Hypotenuse}}$$



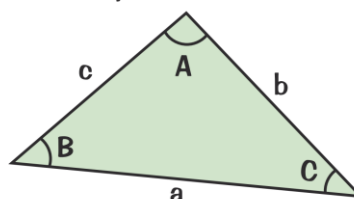
$$\cos x = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan x = \frac{\text{Opposite}}{\text{Adjacent}}$$



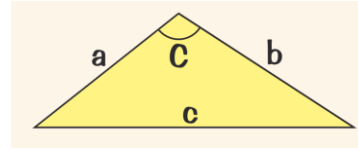
Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



Cosine rule

$$c^2 = a^2 + b^2 - 2ab \cos C$$



Area of a triangle

$$\text{Area of a triangle} = \frac{1}{2}ab \sin C$$

PROBABILITY

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(\text{event happens}) + P(\text{event doesn't happen}) = 1$$

Relative frequency

$$\text{Relative frequency} = \frac{\text{frequency}}{\text{No. of times you tried the experiment}}$$

AND rule

For independent events,

$$P(A \text{ and } B) = P(A) \times P(B)$$

For dependent events,

$$P(A \text{ and } B) = P(A) \times P(B \text{ given that } A \text{ has already occurred})$$

OR rule

For non-mutually exclusive events,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

For mutually exclusive events,

$$P(A \text{ or } B) = P(A) + P(B)$$

STATISTICS

Remember to rearrange data in ascending order!

Mode = most common

Median = middle value

$$\text{Mean} = \frac{\text{total of items}}{\text{Number of items}}$$

Range = highest value – lowest value

Histograms

$$\text{Frequency density} = \frac{\text{Frequency}}{\text{Class width}}$$

Box plots and Cumulative frequency

Inter-quartile range = Upper quartile – Lower quartile

$$IQR = Q_3 - Q_1$$

Loci and Construction

A LOCUS (another ridiculous maths word) is simply:

A LINE or REGION that shows
all the points which fit a given rule.

Make sure you learn how to do these PROPERLY using a ruler and compasses as shown on the next few pages.

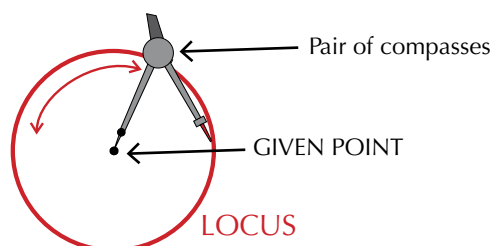
The Four Different Types of Loci



Loci is just the plural of locus.

- 1) The locus of points which are 'A FIXED DISTANCE from a given POINT'.

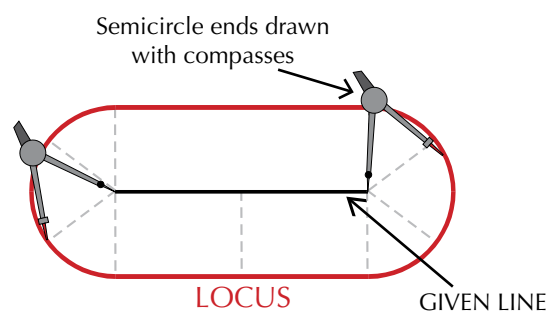
This locus is simply a CIRCLE.



- 2) The locus of points which are 'A FIXED DISTANCE from a given LINE'.

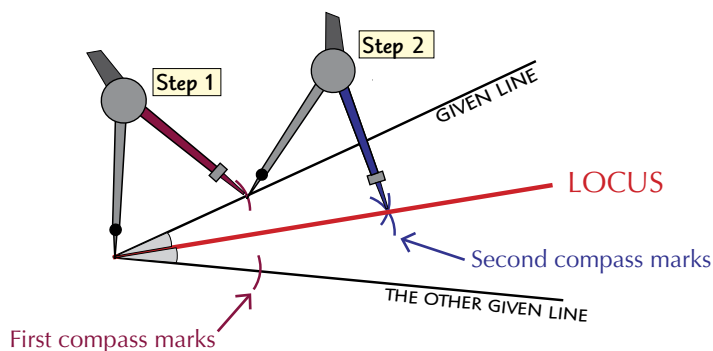
This locus is a SAUSAGE SHAPE.

It has straight sides (drawn with a ruler) and ends which are perfect semicircles (drawn with compasses).



- 3) The locus of points which are 'EQUIDISTANT from TWO GIVEN LINES'.

- 1) Keep the compass setting THE SAME while you make all four marks.
- 2) Make sure you leave your compass marks showing.
- 3) You get two equal angles — i.e. this LOCUS is actually an ANGLE BISECTOR.



- 4) The locus of points which are 'EQUIDISTANT from TWO GIVEN POINTS'.

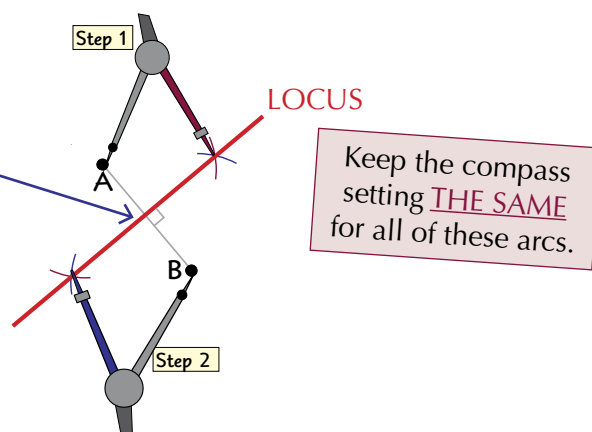
(In the diagram below, A and B are the two given points.)

This LOCUS is all points which are the same distance from A as they are from B.

This time the locus is actually the PERPENDICULAR BISECTOR of the line joining the two points.

The perpendicular bisector of line segment AB is a line at right angles to AB, passing through the midpoint of AB.

This is the method to use if you're asked to draw it.



Keep the compass setting THE SAME for all of these arcs.

Circle Geometry

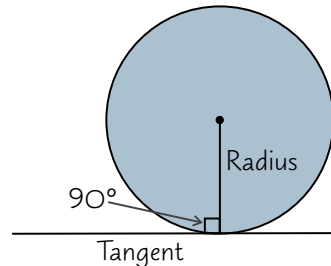
It's time to plunge you into the depths of mathematical peril with a 3-page extravaganza on circle theorems. There's a lot to learn on these pages, but it's all dead useful stuff.

9 Simple Rules to Learn



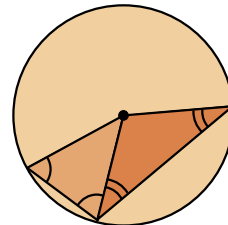
- 1) A TANGENT and a RADIUS meet at 90° .

A TANGENT is a line that just touches a single point on the circumference of a circle. A tangent always makes an angle of exactly 90° with the radius it meets at this point.



- 2) TWO RADII form an ISOSCELES TRIANGLE.

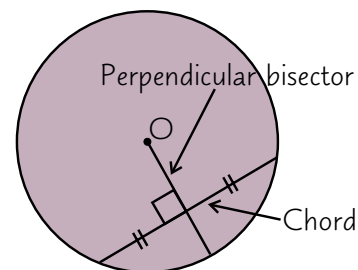
Unlike other isosceles triangles they don't have the little tick marks on the sides to remind you that they are the same — the fact that they are both radii is enough to make it an isosceles triangle.



Radii is the plural of radius.

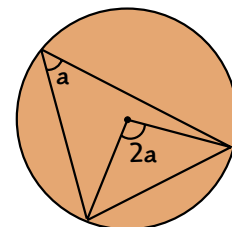
- 3) The PERPENDICULAR BISECTOR of a CHORD passes through the CENTRE of the circle.

A CHORD is any line drawn across a circle. And no matter where you draw a chord, the line that cuts it exactly in half (at 90°), will go through the centre of the circle.



- 4) The angle at the CENTRE of a circle is TWICE the angle at the CIRCUMFERENCE.

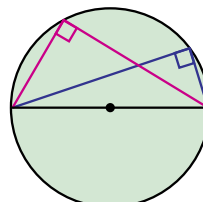
The angle subtended at the centre of a circle is EXACTLY DOUBLE the angle subtended at the circumference of the circle from the same two points (two ends of the same chord).



'Angle subtended at' is just a posh way of saying 'angle made at'.

- 5) The ANGLE in a SEMICIRCLE is 90° .

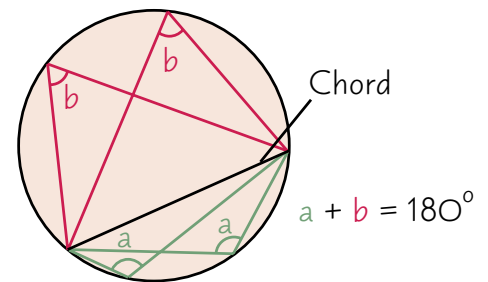
A triangle drawn from the two ends of a diameter will ALWAYS make an angle of 90° where it hits the circumference of the circle, no matter where it hits.



Circle Geometry

6) Angles in the SAME SEGMENT are EQUAL.

All triangles drawn from a chord will have the same angle where they touch the circumference. Also, the two angles on opposite sides of the chord add up to 180° .

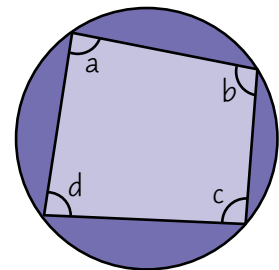


7) OPPOSITE ANGLES in a CYCLIC QUADRILATERAL add up to 180° .

A cyclic quadrilateral is a 4-sided shape with every corner touching the circle. Both pairs of opposite angles add up to 180° .

$$a + c = 180^\circ$$

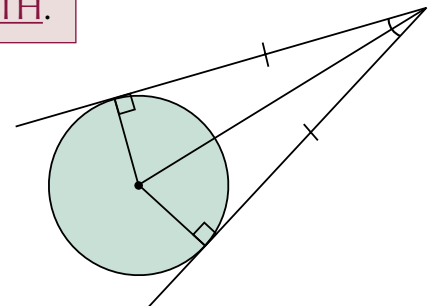
$$b + d = 180^\circ$$



8) TANGENTS from the SAME POINT are the SAME LENGTH.

Two tangents drawn from an outside point are always equal in length, creating two congruent right-angled triangles as shown.

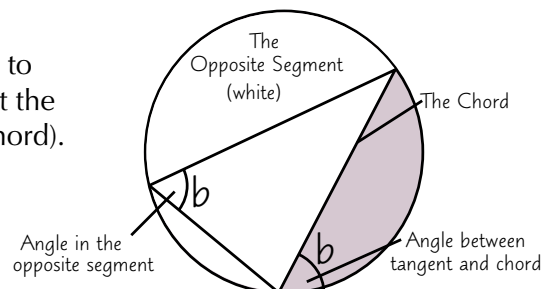
There's more about congruence on p.121.



9) The ALTERNATE SEGMENT THEOREM.

The angle between a tangent and a chord is always equal to 'the angle in the opposite segment' (i.e. the angle made at the circumference by two lines drawn from the ends of the chord).

This is probably the hardest rule, so take care.



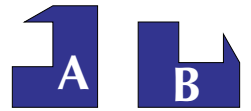
There you have it — nine rules to learn

This page is just more of the same: four more rules to cram into your overcrowded brain. But there's no way round learning this stuff, and once you've learnt all nine, circle geometry becomes much easier.

Congruent Shapes

Congruence is another ridiculous maths word which sounds really complicated when it's not. If two shapes are congruent, they are simply the same — the same size and the same shape. That's all it is. They can however be reflected or rotated.

CONGRUENT
— same size,
same shape



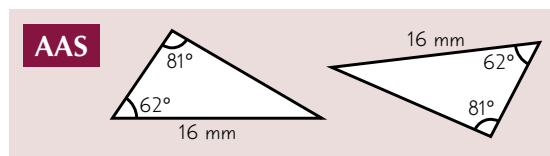
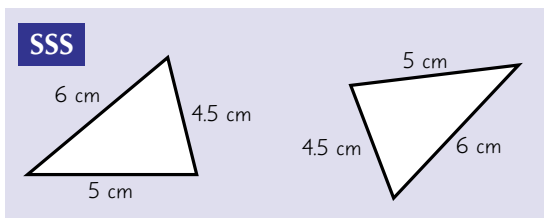
Proving Triangles are Congruent



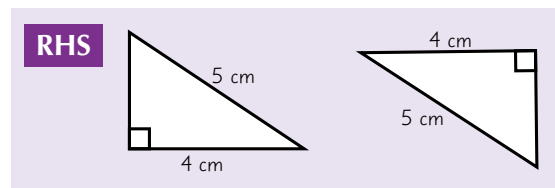
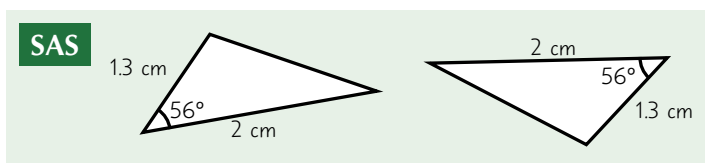
To prove that two triangles are congruent, you have to show that one of the conditions below holds true:

- 1) SSS three sides are the same
- 2) AAS two angles and a corresponding side match up
- 3) SAS two sides and the angle between them match up
- 4) RHS a right angle, the hypotenuse and one other side all match up

The hypotenuse is the longest side of a right-angled triangle — the one opposite the right angle.



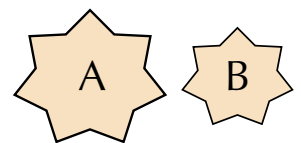
Make sure the sides match up — here, the side is opposite the 81° angle.



Similar Shapes

Similar shapes are exactly the same shape, but can be different sizes (they can also be rotated or reflected).

SIMILAR —
same shape, different size



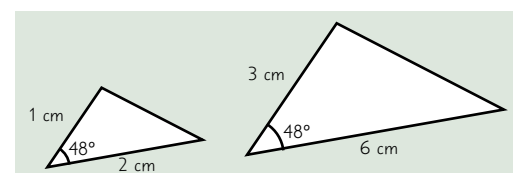
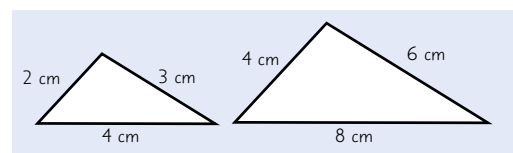
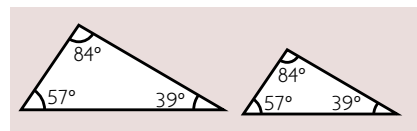
Similar Shapes Have the Same Angles



Generally, for two shapes to be similar, all the angles must match and the sides must be proportional. But for triangles, there are three special conditions — if any one of these is true, you know they're similar.

Two triangles are similar if:

- 1) All the angles match up
i.e. the angles in one triangle are the same as the other.
- 2) All three sides are proportional
i.e. if one side is twice as long as the corresponding side in the other triangle, all the sides are twice as long as the corresponding sides.
- 3) Any two sides are proportional and the angle between them is the same.



Watch out — if one of the triangles has been rotated or flipped over, it might look as if they're not similar, but don't be fooled.