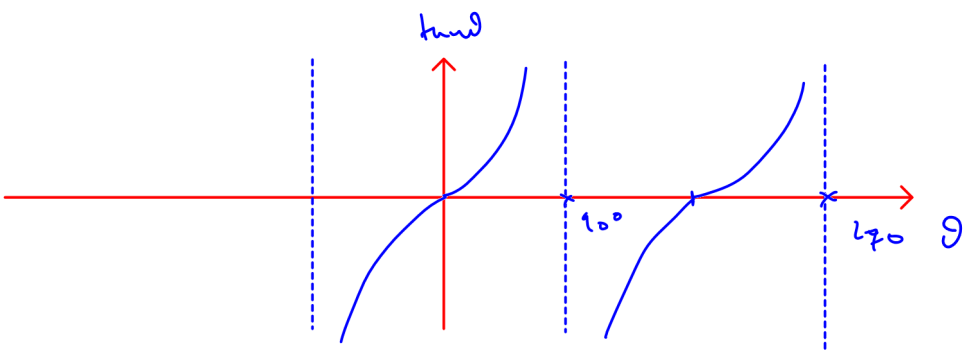


09.10

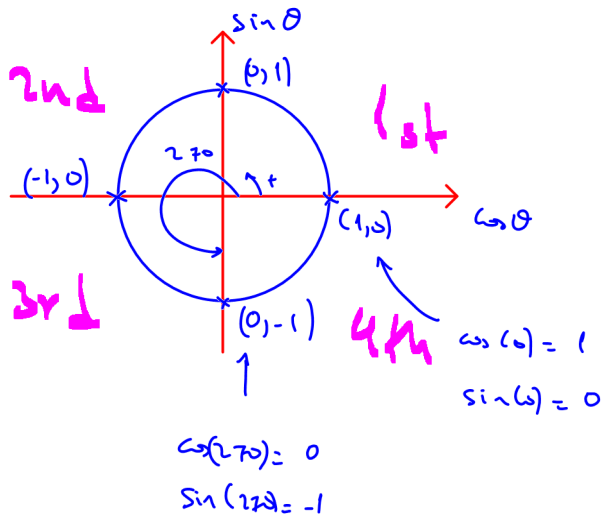
$\tan \theta = \frac{\sin \theta}{\cos \theta}$

$\tan(0) = \frac{\sin(0)}{\cos(0)} = \frac{0}{1} = 0$

$\tan(90) = \frac{\sin 90}{\cos 90} = \frac{1}{0} \Rightarrow \infty$



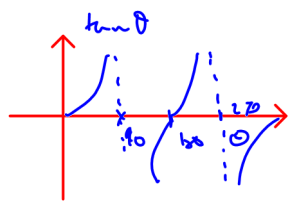
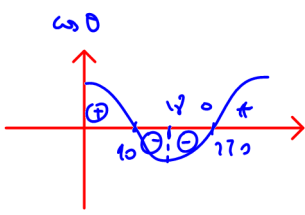
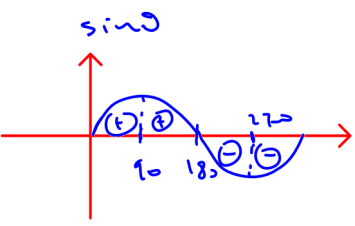
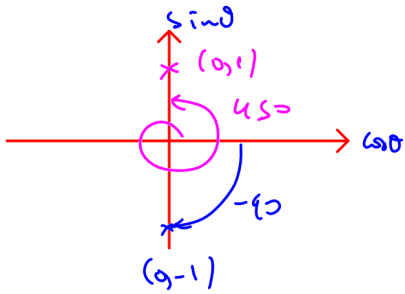
Trigonometric circle
x-axis $\rightarrow \cos \theta$
y-axis $\rightarrow \sin \theta$
radius = 1
anticlockwise = +ve



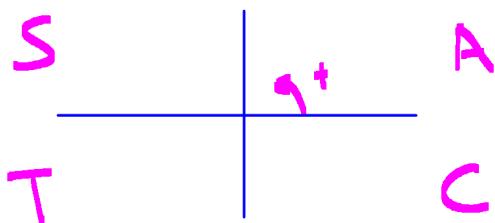
Example 1

Write down the values of:

- a $\sin 90^\circ$
- b $\sin 180^\circ$
- c $\sin 270^\circ$
- d $\cos 180^\circ$
- e $\cos(-90^\circ)$
- f $\cos 450^\circ$



$0 \rightarrow 90^\circ$	$\sin \theta \rightarrow +ve$	$\cos \theta \rightarrow +ve$	$\tan \theta \rightarrow +ve$	(1st)	ASTC
$90 \rightarrow 180^\circ$	$\sin \theta \rightarrow +ve$	$\cos \theta \rightarrow -ve$	$\tan \theta \rightarrow -ve$	(2nd)	
$180 \rightarrow 270^\circ$	$\sin \theta \rightarrow -ve$	$\cos \theta \rightarrow -ve$	$\tan \theta \rightarrow +ve$	(3rd)	
$270 \rightarrow 360$	$\sin \theta \rightarrow -ve$	$\cos \theta \rightarrow +ve$	$\tan \theta \rightarrow +ve$	(4th)	



Example 4

Express in terms of trigonometric ratios of acute angles:

a $\sin(-100^\circ)$

b $\cos 330^\circ$

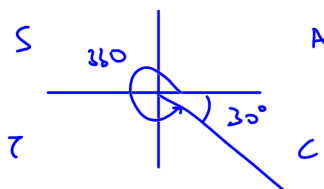
c $\tan 500^\circ$

a) $\sin(-100)$



$\sin(-100) = -\sin 80$

b) $\cos 330$



$\cos 330 = \cos 30$

c $\tan 500^\circ$



$\tan 500 = -\tan 40$

5 Given that θ is an acute angle, express in terms of $\sin \theta$

a $\sin(-\theta)$

b $\sin(180^\circ + \theta)$

c $\sin(360^\circ - \theta)$

d $\sin(-(180^\circ + \theta))$

e $\sin(-180^\circ + \theta)$

f $\sin(-360^\circ + \theta)$

g $\sin(540^\circ + \theta)$

h $\sin(720^\circ - \theta)$

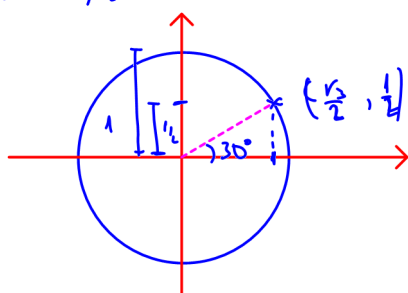
i $\sin(\theta + 720^\circ)$

a) $\sin(-\theta) = -\sin \theta$

b) $\sin(180 + \theta) = -\sin \theta$



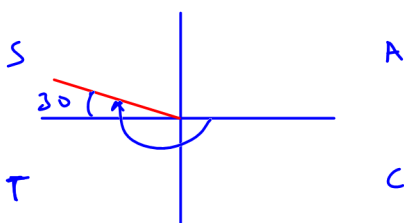
c) $\sin(360 - \theta) = -\sin \theta$



Example 5

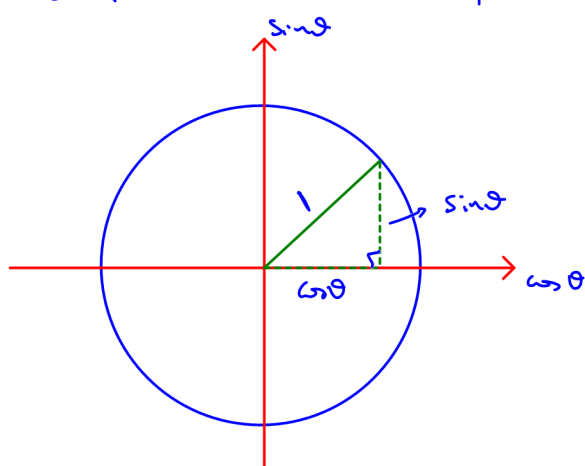
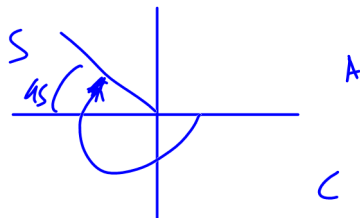
Find the exact value of $\sin(-210^\circ)$.

$$\sin(-210) = + \sin 30 \\ = \frac{1}{2}$$



1 $\tan(-225^\circ)$

$$\tan(-225) = -\tan 45 \\ = -1$$



$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\text{eg. } \theta = 60$$

$$\begin{aligned} &= \sin^2 60 + \cos^2 60 \\ &= (\sin 60)^2 + (\cos 60)^2 \\ &= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \\ &= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \end{aligned}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta}$$

Example 6

Simplify the following expressions:

a $\sin^2 3\theta + \cos^2 3\theta$

b $5 - 5 \sin^2 \theta$

c $\frac{\sin 2\theta}{\sqrt{1 - \sin^2 2\theta}}$

a) 1

b) $\frac{5(1 - \sin^2 \theta)}{5 \cos^2 \theta}$

c) $\frac{\sin 2\theta}{\sqrt{\cos^2 2\theta}} = \frac{\sin 2\theta}{\cos 2\theta} = \tan 2\theta$

Example 7

Prove that $\frac{\cos^4 \theta - \sin^4 \theta}{\cos^2 \theta} \equiv 1 - \tan^2 \theta$

$$\begin{aligned} \text{LHS} \therefore \frac{(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)}{\cos^2 \theta} &= 1 \times \left(\frac{\cos^2 \theta}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} \right) \\ &= 1 - \tan^2 \theta \equiv \text{RHS.} \end{aligned}$$

Example 8

- a Given that $\cos \theta = -\frac{3}{5}$ and that θ is reflex, find the value of $\sin \theta$.
170 → 360° we see that cos θ → -ve (3rd quad.)
- b Given that $\sin \alpha = \frac{2}{5}$ and that α is obtuse, find the exact value of $\cos \alpha$.

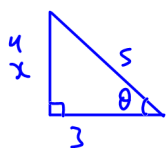
a) on third quad. only $\sin$ is +ve

$$\sin \theta = 1 - \cos \theta$$

$$\sin \theta = -\sqrt{1 - \cos^2 \theta}$$

$$= -\sqrt{1 - \left(-\frac{3}{5}\right)^2}$$

$$= -\frac{4}{5}$$



$$x = \sqrt{5^2 - 3^2}$$

$$= 4$$

$$\sin \theta = -\frac{4}{5}$$

- b Given that $\sin \alpha = \frac{2}{5}$ and that α is obtuse, find the exact value of $\cos \alpha$.

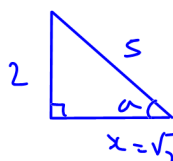
2nd Quad. only $\sin$ +ve

$$\cos \alpha = -\sqrt{1 - \sin^2 \alpha}$$

$$= -\sqrt{1 - \left(\frac{2}{5}\right)^2}$$

$$= -\frac{\sqrt{21}}{5}$$

or



$$x = \sqrt{5^2 - 2^2}$$

$$= \sqrt{21}$$

$$\cos \alpha = -\frac{\sqrt{21}}{5}$$

Example 9

Given that $p = 3 \cos \theta$, and that $q = 2 \sin \theta$, show that $4p^2 + 9q^2 = 36$.

$$= 4 \times (3 \cos \theta)^2 + 9 \times (2 \sin \theta)^2$$

$$= 36 \cos^2 \theta + 36 \sin^2 \theta$$

$$= 36 (\cos^2 \theta + \sin^2 \theta)$$

$$= 36 \times 1$$

$$= 36$$

a $(\sin \theta + \cos \theta)^2 \equiv 1 + 2 \sin \theta \cos \theta$

b $\frac{1}{\cos \theta} - \cos \theta \equiv \sin \theta \tan \theta$

c $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$

d $\cos^2 A - \sin^2 A \equiv 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

e $(2 \sin \theta - \cos \theta)^2 + (\sin \theta + 2 \cos \theta)^2 \equiv 5$

f $2 - (\sin \theta - \cos \theta)^2 \equiv (\sin \theta + \cos \theta)^2$

g $\sin^2 x \cos^2 y - \cos^2 x \sin^2 y \equiv \sin^2 x - \sin^2 y$

a) LHS $\therefore \underbrace{\sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta}_{1 + 2 \sin \theta \cos \theta} = 2(1 + \sin \theta \cos \theta)$

g $\sin^2 x \cos^2 y - \cos^2 x \sin^2 y \equiv \sin^2 x - \sin^2 y$

$$\sin^2 x (1 - \sin^2 y) - (1 - \sin^2 x) \sin^2 y$$

$$\sin^2 x - \cancel{\sin^2 x \sin^2 y} - \sin^2 y + \cancel{\sin^2 x \sin^2 y}$$

$$\sin^2 x - \sin^2 y \equiv \text{RHS}$$

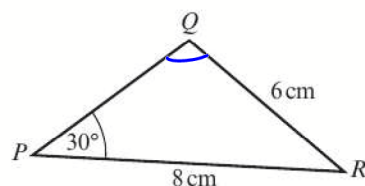
- 12 The diagram shows triangle PQR with $PR = 8$ cm, $QR = 6$ cm and angle $QPR = 30^\circ$.

a Show that $\sin Q = \frac{2}{3}$

(3 marks)

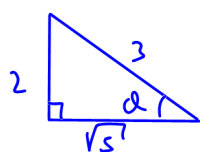
b Given that Q is obtuse, find the exact value of $\cos Q$

(2 marks)



$$c) \frac{\sin Q}{8} = \frac{\sin 30}{6} \quad \sin Q = \frac{8 \times \frac{1}{2}}{6} = \frac{2}{3}$$

b) Since Q is obtuse the quadrant is $\text{QII} \rightarrow -ve$



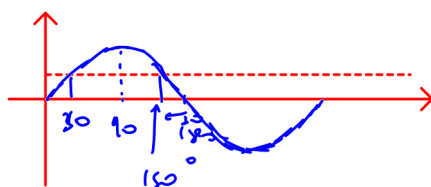
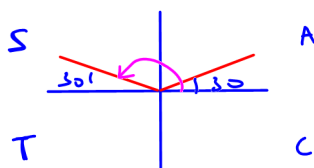
$$\cos Q = -\frac{\sqrt{5}}{3}$$

Example 10

Find the solutions of the equation $\sin \theta = \frac{1}{2}$ in the interval $0 \leq \theta \leq 360^\circ$. $\hookrightarrow$

$\sin \theta \rightarrow +ve$ (1st & 2nd)

$$\sin \theta = \frac{1}{2}, \theta = \sin^{-1}(\frac{1}{2}) = 30, 180 - 30 = 30^\circ, 150^\circ$$

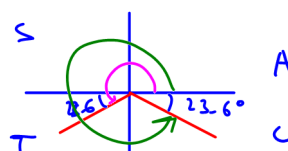


Example 11

Solve, in the interval $0 \leq x \leq 360^\circ$, $5 \sin x = -2$.

$\sin x \rightarrow -ve$ (3rd & 4th)

$$\sin x = -\frac{2}{5} \Rightarrow x = 25.6, 180 + 25.6, 360 - 25.6$$



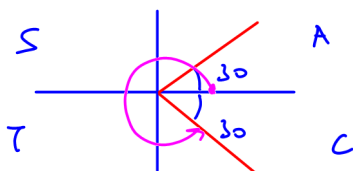
Example 12

Solve, in the interval $0 < x \leq 360^\circ$, $\cos x = \frac{\sqrt{3}}{2}$

$$\cos x = \frac{\sqrt{3}}{2}$$

$$x = 30, 360 - 30$$

$\cos$ (pos) (1st & 4th)

**Example 13**

Find the values of θ in the interval $0 < \theta \leq 360^\circ$ that satisfy the equation $\sin \theta = \sqrt{3} \cos \theta$.

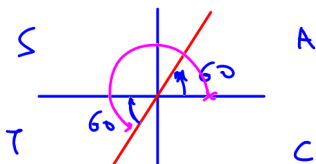
The issue: we have two trigon. ratios

we can only solve by having one trig. ratio.

$$\frac{\sin \theta}{\cos \theta} = \frac{\sqrt{3} \cos \theta}{\cos \theta} \Rightarrow \tan \theta = \sqrt{3} \rightarrow \tan \theta \text{ (pos)} \text{ (1st \& 3rd)}$$

$$\tan \theta = \sqrt{3}$$

$$\theta = 60, 180 + 60$$



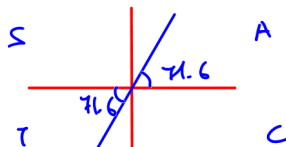
- 9 Find all the values of θ , to 1 decimal place, in the interval $0 < \theta < 360^\circ$ for which $\tan^2 \theta = 9$. (5 marks)

$$\tan \theta = \pm 3$$

$$\tan \theta = 3$$

(1st & 3rd)

$$\theta = 71.6^\circ, 180 + 71.6$$



$$\tan \theta = -3$$

(2nd & 4th)

$$\theta = -71.6, 180 - 71.6, 360 - 71.6$$

