

Algebraic Proof

1 Prove that $(n + 5)^2 - (3n + 8) = (n + 3)(n + 4) + 5$

(Total for Question 1 is 3 marks)

2 Prove that $(2n - 1)^2 - (n - 3)^2 = (3n - 1)(n + 1) - 7$

(Total for Question 2 is 3 marks)

3 Prove that $(3n - 5)^2 - 2(4n - 5)(n - 3) = (n + 5)(n - 1)$

(Total for Question 3 is 3 marks)

4 Prove that $(n - 3)^2 - (15 + n)(15 - n) = 2(n - 12)(n + 9)$

(Total for Question 4 is 3 marks)

5 n is an integer such that $n > 3$

Prove algebraically that $(n - 2)^2 - (n - 5)^2$ is always a multiple of 3.

(Total for Question 5 is 3 marks)

6 n is a positive integer.

Prove algebraically that $(3n + 1)^2 - (3n - 4)^2$ is always a multiple of 15.

(Total for Question 6 is 3 marks)

7 n is a positive integer.

Prove algebraically that $(2n + 5)^2 - (2n + 1)^2$ is always a multiple of 8

(Total for Question 7 is 3 marks)

8 n is a positive integer such that $n > 2$

Prove algebraically that $(2n + 3)^2 + (3 - n)^2 - (n + 5)^2$ is always one more than a multiple of 4.

(Total for Question 8 is 4 marks)

9 Prove algebraically that the sum of five consecutive positive integers is always a multiple of 5.

(Total for Question 9 is 2 marks)

10 Arjan says: “The sum of four consecutive positive integers is always a multiple of 4”.

Use an algebraic method to prove that Arjun is incorrect.

(Total for Question 10 is 2 marks)

11 Prove algebraically that the sum of six consecutive positive integers is always a multiple of 3.

(Total for Question 11 is 2 marks)

12 n is a positive integer.

Prove that $(4n - 3)^2 - 3(5n - 3)(n - 1)$ is always a square number.

(Total for Question 12 is 3 marks)

13 n is a positive integer.

Prove that $(3n + 1)(9n^2 - 3n + 1)$ is always 1 more than a cube number.

(Total for Question 13 is 4 marks)

14 n is a positive integer.

Prove that $(n + 2)^3 - n^3$ is always even.

(Total for Question 14 is 4 marks)

15 n is an integer.

Prove that $n^2 - 6n + 10$ is always positive.

(Total for Question 15 is 3 marks)

16 n is an integer.

Prove that $n^2 + 3n + 3$ is always positive.

(Total for Question 16 is 3 marks)

17 n is an integer.

Prove that $2n - n^2 - 2$ is always negative.

(Total for Question 17 is 3 marks)

18 n and m are consecutive integers and $m > n$.

Prove algebraically that $m^2 - n^2$ is always an odd number.

(Total for Question 18 is 3 marks)

19 n and m are consecutive integers and $m > n$.

Prove algebraically that $mn + m$ is always a square number.

(Total for Question 19 is 3 marks)

20 Prove algebraically that the sum of three consecutive even numbers is always a multiple of 6.

(Total for Question 20 is 2 marks)

21 Prove algebraically that the difference between the squares of two consecutive even numbers is always a multiple of 4

(Total for Question 21 is 3 marks)

22 Prove algebraically that the sum of the squares of three consecutive integers is one less than a multiple of 3.

(Total for Question 22 is 4 marks)

23 Prove algebraically that the difference between the squares of consecutive integers is equal to the sum of the two integers.

(Total for Question 23 is 3 marks)

24 Prove algebraically that the product of two consecutive odd numbers is one less than a multiple of 4.

(Total for Question 24 is 3 marks)

25 Prove algebraically that the product of three consecutive even numbers is always a multiple of 8.

(Total for Question 25 is 3 marks)

- 26** a and b are positive integers.
 a is two more than a multiple of 5.
 b is two less than a multiple of 5.

Prove that ab is one more than a multiple of 5.

(Total for Question 26 is 4 marks)

- 27** Prove that the sum of the squares of three consecutive integers is equal to five more than three times the product of the largest and smallest of the three integers.

(Total for Question 27 is 3 marks)

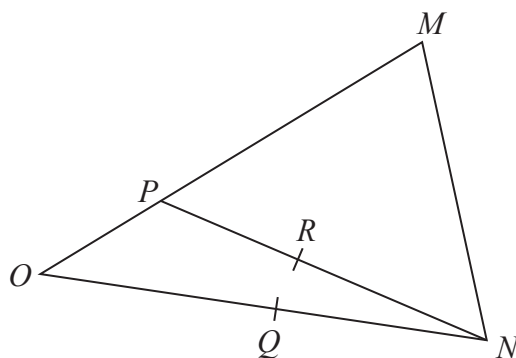


Diagram **NOT**
accurately drawn

OMN is a triangle.

P is the point on OM such that $OP = \frac{1}{4} OM$

Q is the midpoint of ON

R is the midpoint of PN

$$\vec{OP} = \mathbf{p} \quad \vec{OQ} = \mathbf{q}$$

(a) Find, in terms of $\mathbf{p}$ and $\mathbf{q}$,

(i) $\vec{MN}$

(ii) $\vec{PR}$

(2)

(b) Use a vector method to prove that QR is parallel to OP

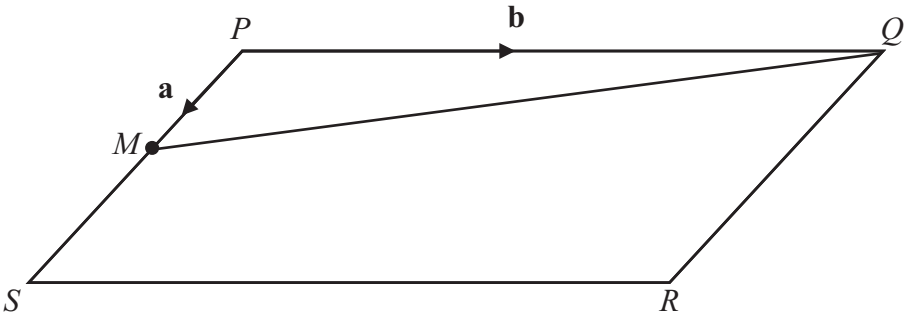
(2)

The diagram shows a parallelogram, $PQRS$.

M is the midpoint of PS .

$\overrightarrow{PM} = \mathbf{a}$ $\overrightarrow{PQ} = \mathbf{b}$

Diagram **NOT**
accurately drawn



(a) Find, in terms of $\mathbf{a}$ and/or $\mathbf{b}$,

(i) $\overrightarrow{PS}$

.....

(ii) $\overrightarrow{PR}$

.....

(iii) $\overrightarrow{MQ}$

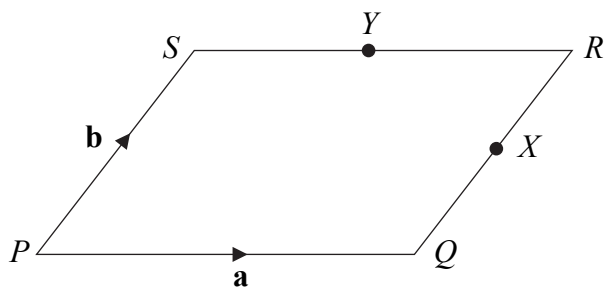
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(3)

N is the point on MQ such that $MN = \frac{1}{3}MQ$

(b) Use a vector method to prove that PNR is a straight line.

(2)



$PQRS$ is a parallelogram.

X is the midpoint of QR and Y is the midpoint of SR .

$\vec{PQ} = \mathbf{a}$ and $\vec{PS} = \mathbf{b}$.

(a) Write down, in terms of $\mathbf{a}$ and $\mathbf{b}$, expressions for

(i) $\vec{PX}$

.....

(ii) $\vec{PY}$

.....

(iii) $\vec{QS}$

.....

(3)

(b) Use a vector method to show that XY is parallel to QS and that $XY = \frac{1}{2}QS$.

(2)

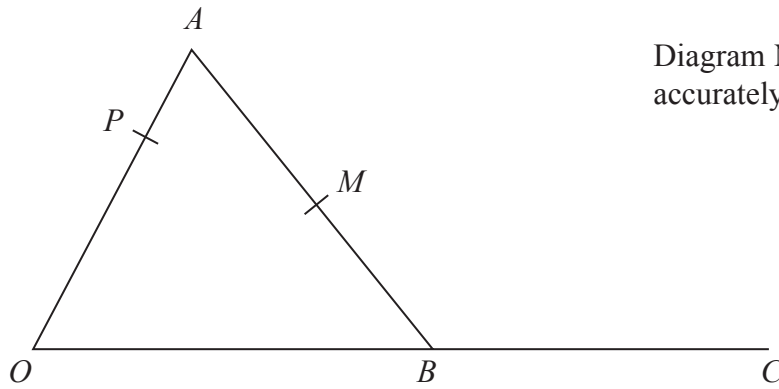


Diagram **NOT**
accurately drawn

OAB is a triangle.

P is the point on OA such that $OP:PA = 2:1$

C is the point such that B is the midpoint of OC .

M is the midpoint of AB .

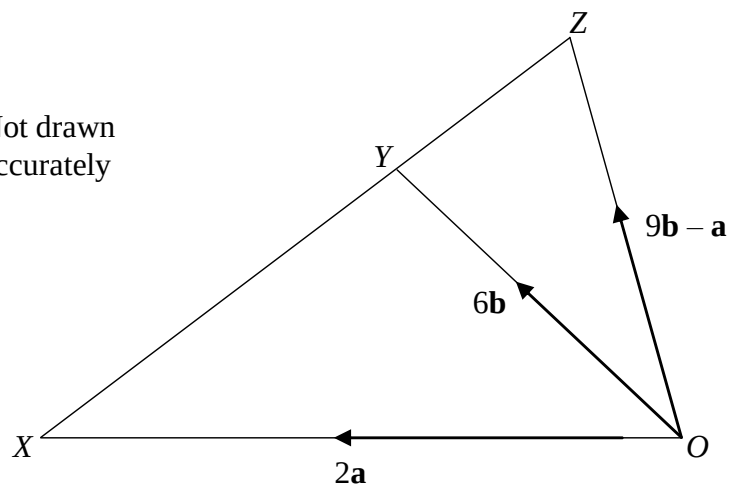
$$\vec{OA} = 6\mathbf{a}$$

$$\vec{OB} = 4\mathbf{b}$$

Show that PMC is a straight line.

7

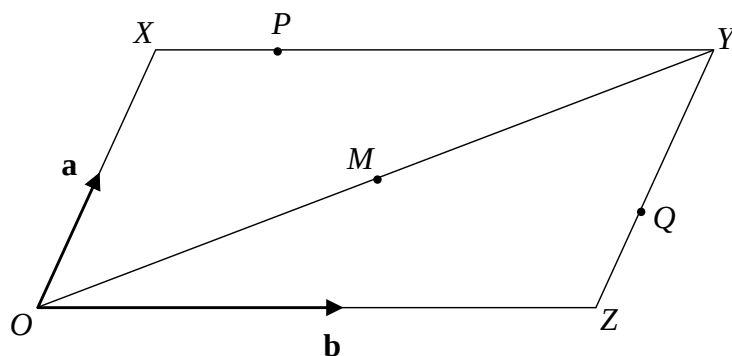
Not drawn
accurately



Prove, using vectors, that XYZ is a straight line.

(Total for Question 7 is 3 marks)

8 $OXYZ$ is a parallelogram



$$\vec{OX} = \mathbf{a} \quad \vec{OZ} = \mathbf{b}$$

$$XP : PY = 1 : 3$$

$$ZQ : QY = 2 : 3$$

M is the midpoint of OY

(a) Write $\vec{PQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

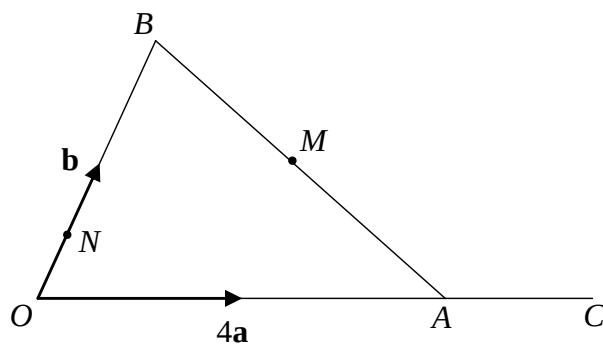
(b) Write $\vec{MQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(2)

(3)

(Total for Question 8 is 5 marks)

9



$$\vec{OA} = 4\mathbf{a} \quad \vec{OB} = \mathbf{b}$$

$$OA : OC = 3 : 4$$

$$ON : OB = 2 : 9$$

M is the midpoint of AB

(a) Write $\vec{MC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

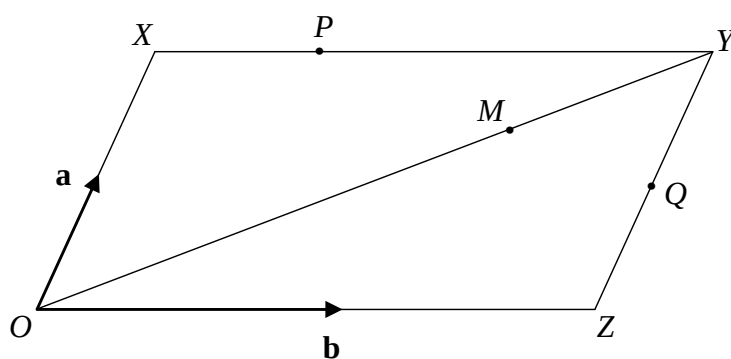
(b) Write $\vec{NM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

(3)

(2)

(Total for Question 9 is 5 marks)

10 $OXYZ$ is a parallelogram



$$\vec{OX} = \mathbf{a} \quad \vec{OZ} = \mathbf{b}$$

$$ZQ = QY$$

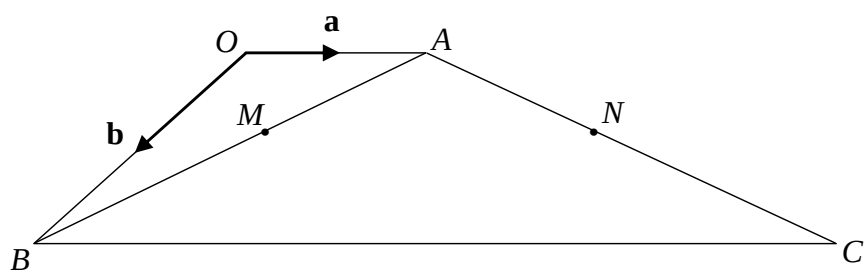
$$XP : PY = 1 : 2$$

$$OM : MY = 5 : 2$$

Prove, using vectors, that PMQ is a straight line.

(Total for Question 10 is 4 marks)

11 $OACB$ is a trapezium



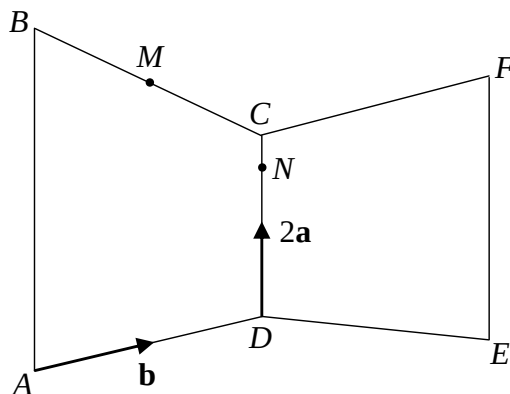
$$\vec{OA} = \mathbf{a} \quad \vec{OB} = \mathbf{b}$$

M and N are the midpoints of AB and AC .

Prove, using vectors, that MN is parallel to OA .

(Total for Question 11 is 4 marks)

12 $ABCD$ and $CDEF$ are trapeziums



$$\vec{DC} = 2\mathbf{a} \quad \vec{AD} = \vec{CF} = \mathbf{b}$$

$$AB : DC : EF = 4 : 2 : 3$$

M is the midpoint of BC .

N is on the line CD .

MNE is a straight line.

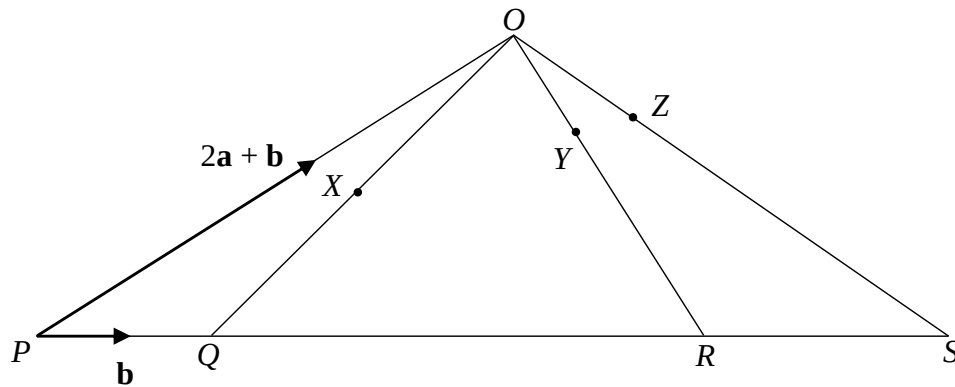
$DN : NC = k : 1$, where k is an integer.

Work out the value of k .

$$k = \dots\dots\dots$$

(Total for Question 12 is 5 marks)

13 POS is a triangle.



$$\overrightarrow{PQ} = \mathbf{b} \quad \overrightarrow{PO} = 2\mathbf{a} + \mathbf{b}$$

X is the midpoint of QO

$$OY : YR = 1 : 2$$

$$PQ : QR : RS = 2 : 6 : 3$$

XYZ is a straight line.

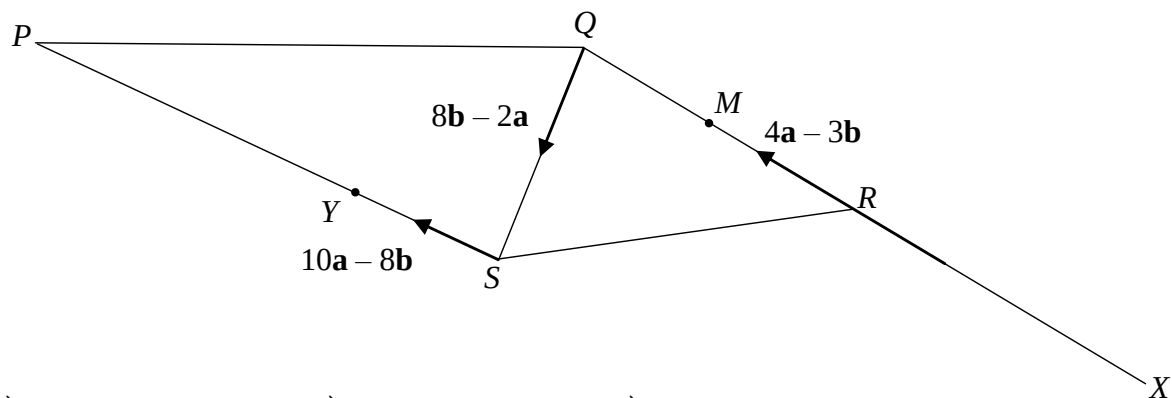
$$OZ : OS = 1 : k$$

Work out the value of k .

$$k = \dots\dots\dots$$

(Total for Question 13 is 6 marks)

14 PQRS is a quadrilateral.



$$\vec{SY} = 10\mathbf{a} - 8\mathbf{b}$$

$$\vec{QS} = 8\mathbf{b} - 2\mathbf{a}$$

$$\vec{RM} = 4\mathbf{a} - 3\mathbf{b}$$

$$RM = MQ$$

$$SY : YP = 1 : 2$$

QRX is a straight line.

XS is parallel to RP.

Work out XS : RP

Give your answer in the form $n : 1$

(Total for Question 14 is 6 marks)