

CHAPTER 11

11.1 Vectors

- a** $2\mathbf{b}$ **b** $-2\mathbf{a}$ **c** $-\mathbf{b}$
d $\mathbf{a} + \mathbf{b}$ **e** $\mathbf{a} - \mathbf{b}$ **f** $-2\mathbf{a} + 2\mathbf{b}$
g $2\mathbf{a} - \mathbf{b}$ **h** $2\mathbf{a} - \mathbf{b}$
- a** $-\mathbf{a} + \mathbf{b}$ **b** $\mathbf{a} - \mathbf{b} + \mathbf{d}$
c $-\mathbf{b} + \mathbf{c}$ **d** $\mathbf{a} - \mathbf{c} + \mathbf{d}$
- a** $\mathbf{a} + \mathbf{b}$ **b** $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$ **c** $\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$
- $-\frac{4}{5}$
- $-\frac{7}{20}\mathbf{a} - \frac{2}{5}\mathbf{b}$
- $\overrightarrow{AC} = \mathbf{a} + \mathbf{b}$, $\overrightarrow{DE} = \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} = \frac{1}{2}\overrightarrow{AC}$, so the two lines are parallel as one vector is a scalar multiple of the other.
- a** $\overrightarrow{MN} = \frac{1}{2}\mathbf{b} + k\mathbf{a} - \frac{1}{2}\mathbf{c}$ and $\overrightarrow{MN} = -\frac{1}{2}\mathbf{b} + \mathbf{a} + \frac{1}{2}\mathbf{c}$
 Adding gives $2\overrightarrow{MN} = k\mathbf{a} + \mathbf{a}$, so $\overrightarrow{MN} = \left(\frac{1+k}{2}\right)\mathbf{a}$
b $\overrightarrow{EH}$, $\overrightarrow{FG}$ and $\overrightarrow{MN}$ are all scalar multiples of $\mathbf{a}$, so are parallel to each other.

11.2 Representing vectors

- a** $-21\mathbf{i} + 9\mathbf{j}$ **b** $-11\mathbf{i} + 13\mathbf{j}$ **c** $13\mathbf{i} - 18\mathbf{j}$
- a** $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ **b** $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ **c** $\begin{pmatrix} 12 \\ -13 \end{pmatrix}$
- $-3\mathbf{i} + 2\mathbf{j}$
- $19\mathbf{i} - 8\mathbf{j}$
- $p = -19$, $q = -8$
- $2\mathbf{a} + 5\mathbf{b} = 11\mathbf{i} + 4\mathbf{j}$, $2\mathbf{d} - \mathbf{c} = 5\mathbf{i} + 2\mathbf{j}$, so the vectors are not parallel
- $p = 7$, $q = -1$

11.3 Magnitude and direction

- a** 10 **b** $\sqrt{41}$ **c** $3\sqrt{5}$ **d** $8\sqrt{2}$
- a** 63.4° above (1 d.p.) **b** 171.9° below (1 d.p.)
c 90° above **d** 45° below
- $-\frac{5}{13}\mathbf{i} + \frac{12}{13}\mathbf{j}$
- $25\sqrt{2}\mathbf{i} - 25\sqrt{2}\mathbf{j}$
- $p = 3.76$, $q = 10.3$ (3 s.f.)
- $p = \pm 7$
- a** 38.7° (1 d.p.) **b** 74.1° (1 d.p.) **c** 112.7° (1 d.p.)
- $p = -6$ and $q = \pm 2.5$

11.4 Position vectors

- a** $-2\mathbf{i} + 3\mathbf{j}$ **b** $7\mathbf{i}$ **c** $9\mathbf{i} - 3\mathbf{j}$
- a** $6\mathbf{i} - 2\mathbf{j}$ **b** $2\mathbf{i} + 4\mathbf{j}$ **c** $5\mathbf{i} - 5\mathbf{j}$
- a** 10 **b** $3\sqrt{10}$ **c** $\sqrt{34}$
- a** $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$ **b** $\sqrt{41}$
- $\frac{22}{5}\mathbf{i} + \frac{29}{5}\mathbf{j}$ and $-2\mathbf{i} - 7\mathbf{j}$

11.5 Solving geometric problems

- a** $\mathbf{q} - \mathbf{p}$ **b** $\overrightarrow{PN} = \frac{1}{4}\overrightarrow{PQ}$
c $\overrightarrow{ON} = \overrightarrow{OP} + \overrightarrow{PN}$ **d** $\frac{3}{4}\mathbf{p} + \frac{1}{4}\mathbf{q}$
- $\frac{3}{5}\mathbf{a} + \frac{2}{5}\mathbf{b}$
- $\overrightarrow{XY} = -\frac{3}{10}\mathbf{a} + \frac{2}{5}\mathbf{b}$
- 78.7°
- a** $\overrightarrow{KL} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{PR} = \frac{1}{2}\mathbf{b} - \frac{1}{2}\mathbf{a}$, $\overrightarrow{PR} = \frac{1}{2}\overrightarrow{KL}$
b Any two vectors, where one is a scalar multiple of the other, are parallel.
- a** i $\frac{1}{3}\mathbf{a} + \mathbf{b}$ ii $-\mathbf{a}$
b $\overrightarrow{AM} = \overrightarrow{AW} + \overrightarrow{WM} = \overrightarrow{AW} + \lambda\overrightarrow{WY} = \frac{1}{3}\mathbf{a} + \lambda\left(\frac{1}{3}\mathbf{a} + \mathbf{b}\right)$
 $= \frac{1}{3}(1 + \lambda)\mathbf{a} + \lambda\mathbf{b}$
c $\overrightarrow{AM} = \overrightarrow{AB} + \overrightarrow{BX} + \overrightarrow{XM} = \mathbf{a} + \frac{1}{2}\mathbf{b} - \mu\mathbf{a} = (1 - \mu)\mathbf{a} + \frac{1}{2}\mathbf{b}$
d Equating coefficients of $\mathbf{a}$ and $\mathbf{b}$, $\mu + \frac{1}{3}\lambda = \frac{2}{3}$ and $\lambda = \frac{1}{2}$
 So $\mu = \frac{2}{3} - \frac{1}{6} = \frac{1}{2} \Rightarrow M$ is the midpoint of WY and XZ .
- a** $\overrightarrow{PQ} = \frac{k}{1+k}\overrightarrow{OA} - \frac{k}{1+k}\overrightarrow{BA} = \frac{k}{1+k}\overrightarrow{OB}$. So $\overrightarrow{PQ}$ is a scalar multiple of $\overrightarrow{OB}$ and these vectors are parallel.
b $k = \frac{1}{2}$

11.6 Modelling with vectors

- a** 13 m s^{-1} **b** 10 km s^{-1} **c** $\sqrt{37}\text{ mm s}^{-1}$ **d** 14 cm s^{-1}
- a** 5 m s^{-1} **b** 20 m **c** 5.5 seconds
- $\left(\frac{3}{2}\mathbf{i} - \frac{9}{2}\mathbf{j}\right)\text{ km s}^{-2}$
- a** $(9\mathbf{i} - 16\mathbf{j})\text{ m s}^{-1}$ **b** $\sqrt{337}\text{ m s}^{-1}$
- a** $(-3\mathbf{i} + 6\mathbf{j})\text{ km}$ **b** $\sqrt{137}\text{ km}$ **c** 110.0° (1 d.p.)
- a** 56.3° (1 d.p.) below the positive x -axis
b Solve $3 + p = 2k$ and $q - 7 = -3k$ by eliminating k .
c $3\sqrt{13}\text{ N}$
- 16.5 km^2
- a** 76.0° **b** $a = 11$, $b = 25$ **c** $c = -8$

Problem solving: Set A

- B** **a** $7\mathbf{i} - 7\mathbf{j}$ **b** $AB = 13$, $AC = 13$, $BC = 7\sqrt{2}$ **c** 44.8°
S **a** $7\mathbf{i} + \mathbf{j}$ **b** 44.0° **c** 13.5
G 26

Problem solving: Set B

- B** **a** $\overrightarrow{OR} = \overrightarrow{OA} + \overrightarrow{AR} = \mathbf{a} + \frac{3}{5}(-\mathbf{a} + \mathbf{b}) = \frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}$
 $\overrightarrow{OP} = 5\overrightarrow{OR} = 5\left(\frac{2}{5}\mathbf{a} + \frac{3}{5}\mathbf{b}\right) = 2\mathbf{a} + 3\mathbf{b}$
b $\overrightarrow{QB} = \mathbf{a} + \mathbf{b}$, $\overrightarrow{BP} = 2\mathbf{a} + 2\mathbf{b}$, $\overrightarrow{QB} = \frac{1}{2}\overrightarrow{BP}$ so lines are parallel. As B is a shared point, Q , B and P are collinear.
a $\frac{3}{5}$ **b** $\overrightarrow{OC} = 3\mathbf{b}$
G Let $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AD} = \mathbf{b}$. Let point of intersection of PQ and MC be R , and let $MR = kMC$ and $PR = jPQ$. So going via M ,
 $\overrightarrow{AR} = \frac{1}{2}\mathbf{b} + k\left(\frac{1}{2}\mathbf{b} + \mathbf{a}\right) = k\mathbf{a} + \left(\frac{1}{2}k + \frac{1}{2}\right)\mathbf{b}$
 And going via P ,
 $\overrightarrow{AR} = \frac{4}{5}\mathbf{a} + j\left(\mathbf{b} - \frac{2}{5}\mathbf{a}\right) = \left(\frac{4}{5} - \frac{2}{5}j\right)\mathbf{a} + j\mathbf{b}$
 Equating coefficients of $\mathbf{a}$ and $\mathbf{b}$, $k = \frac{4}{5} - \frac{2}{5}j$ and $j = \frac{1}{2}k + \frac{1}{2}$
 which gives $k = \frac{1}{2}$