

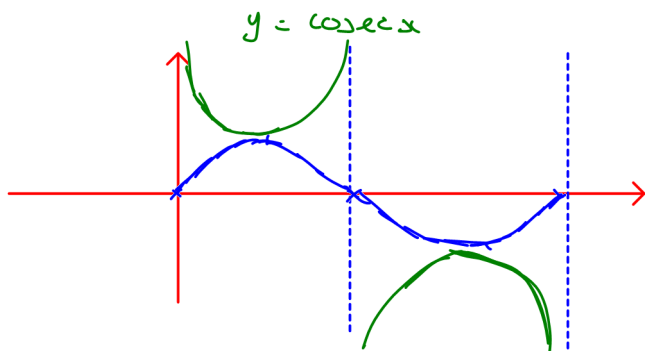
28.04

$$y = \operatorname{cosec} x$$

$$y = \frac{1}{\sin x}$$

$$x=0 \quad y = \frac{1}{\sin 0} = \frac{1}{0} \rightarrow \infty$$

$$x = \frac{\pi}{2} \quad y = \frac{1}{\sin \frac{\pi}{2}} = 1$$



4 a Prove that $\frac{\cot x + \tan x}{\sec x} \equiv \operatorname{cosec} x$ (4 marks)

b Hence solve in the interval $0 \leq x \leq 360^\circ$, the equation $\frac{\cot x + \tan x}{\sec x} = 4$ (3 marks)

a)

$$\text{LHS} : \left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x} \right) \div \frac{1}{\cos x} = \frac{\cos^2 x}{\sin x} + \sin x$$

$$= \frac{\cos^2 x}{\sin x} + \frac{\sin^2 x}{\sin x} = \frac{\cos^2 x + \sin^2 x}{\sin x} = \frac{1}{\sin x} = \operatorname{cosec} x$$

$$b) \operatorname{cosec} x = 4 \Rightarrow \sin x = \frac{1}{4}$$

5 Solve $\sec^2 x = 5 + 3 \tan x$ in the interval $0 \leq x \leq 360^\circ$, giving your answers to 1 decimal place where necessary. (4 marks)

$$\sec^2 x = 1 + \tan^2 x$$

$$\tan^2 x + 1 = 5 + 3 \tan x$$

$$\tan^2 x - 3 \tan x - 4 = 0$$

$$(\tan x - 4)(\tan x + 1) = 0$$

$$\tan x = 4 \quad (1st \& 3rd)$$

$$\tan x = -1 \quad (2nd \& 4th)$$

7 Given that $\arccos(x-3) = -\frac{\pi}{4}$, find the exact value of x .

$$x-3: \cos\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$x = \frac{\sqrt{2}}{2} + 3 = \frac{\sqrt{2} + 6}{2}$$

Given that $\frac{1 - \tan^2 x}{1 + \tan^2 x} = z$, show that $\operatorname{cosec}^2 x = \frac{2}{1-z}$, $z \neq 1$.

(5 marks)

$$\frac{1 - \frac{\sin x}{\cos x}}{1 + \frac{\sin^2 x}{\cos^2 x}} = \frac{\frac{\cos^2 x - \sin^2 x}{\cos^2 x}}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}} \Rightarrow \frac{\cos^2 x - \sin^2 x}{1} = z$$

$$1 - \sin^2 x - \sin^2 x = z$$

$$1 - 2\sin^2 x = z$$

$$\sin^2 x = \frac{1-z}{2} \Rightarrow \operatorname{cosec}^2 x = \frac{2}{1-z}$$

4 Express $\frac{\cos(x-y)}{\sin(x+y)}$ in terms of $\tan x$ and $\tan y$.

(3 marks)

$$\frac{\cos x \cos y + \sin x \sin y}{\sin x \cos y + \cos x \sin y} \div \cos x \cos y$$

$$\frac{\frac{\cos x \cos x}{\cos x \cos y} + \frac{\sin x \sin y}{\cos x \cos y}}{\frac{\sin x \cos y}{\cos x \cos y} + \frac{\cos x \sin y}{\cos x \cos y}} = \frac{1 + \tan x \tan y}{\tan x + \tan y}$$

7 a Use $\sin(x + \alpha) \equiv \sin x \cos \alpha + \cos x \sin \alpha$ to show that $\sin 105^\circ = \frac{\sqrt{6} + \sqrt{2}}{4}$

(4 marks)

b Hence, or otherwise, show that $\operatorname{cosec} 105^\circ = \sqrt{6} - \sqrt{2}$

(3 marks)

$$\begin{aligned} \text{a) } \sin(60+45) &= \sin 60 \cos 45 + \cos 60 \sin 45 \\ &= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} + \frac{1}{2} \times \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4} \end{aligned}$$

5 a Show that $\cos^4 x - \sin^4 x \equiv \cos 2x$.

(2 marks)

b Hence find the exact value of $\cos^4 \frac{\pi}{12} - \sin^4 \frac{\pi}{12}$

(2 marks)

$$\text{a) LHS: } (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$$

$$1 \times \cos 2x = \cos 2x$$

$$\begin{aligned} \text{b) } \cos\left(2 \times \frac{\pi}{12}\right) &= \cos \frac{\pi}{6} \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

- 6 Given that $a = 2 \sin x$, $b = 2 \cos x$, $c = \cos 2x$ and $d = \tan 2x$,
- a express c in terms of a (2 marks)
- b show that $d = \frac{2ab}{b^2 - a^2}$ (3 marks)

$\sin x = \frac{a}{2}$

a) $c = 1 - 2 \sin^2 x$
 $= 1 - 2 \left(\frac{a}{2}\right)^2$
 $c = 1 - \frac{a^2}{2} = \frac{2 - a^2}{2} = c$

b) $\tan x = \frac{\sin x}{\cos x}$
 $= \frac{2 \sin x \cos x}{c}$

$d = \frac{2 \left(\frac{a}{2}\right) \left(\frac{b}{2}\right)}{c}$

$\sin 2x = 2 \sin x \cos x$

- 4 a Show that $\sin(x - 45^\circ) = \cos x$ can be written as $\tan x = a + \sqrt{b}$, where a and b are real constants to be found. (3 marks)
- b Hence, or otherwise, solve $\sin(x - 45^\circ) = \cos x$ in the interval $0 \leq x \leq 360^\circ$, giving your answers to 1 decimal place. (3 marks)

a) $\sin x \cos 45^\circ - \cos x \sin 45^\circ = \cos x$
 $\frac{1}{2} \sin x - \frac{\sqrt{2}}{2} \cos x = \cos x \quad (\div \cos x)$
 $\frac{\sqrt{2}}{2} \tan x - \frac{\sqrt{2}}{2} = 1 \quad \tan x = \frac{1 + \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$
 $= \frac{2 + \frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} \times \frac{2}{\sqrt{2}}$
 $= \frac{2 + \sqrt{2}}{\sqrt{2}}$

- 6 a Given that $\tan 2x - \tan x = 0$, show that $\tan x + \tan^3 x = 0$. (3 marks)
- b Hence solve $\tan 2x = \tan x$ in the interval $0 \leq x \leq 2\pi$. Give your answers in terms of π . (3 marks)

a) $\frac{2 \tan x}{1 - \tan^2 x} - \tan x = 0 \quad \frac{2 \tan x - \tan x (1 - \tan^2 x)}{1 - \tan^2 x} = 0$

$2 \tan x - \tan x + \tan^3 x = 0$
 $\tan x + \tan^3 x = 0$

b) $\tan x (1 + \tan^2 x) = 0$
 $\tan x = 0 \rightarrow \tan^2 x = -1$ no solution

4 $h(\theta) = 2 \sin \theta + 4 \cos \theta$

a Given that $h(\theta) = R \sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$, find the exact value of R and the value of α to 2 decimal places. (3 marks)

b Hence solve $2 \sin \theta + 4 \cos \theta = 3$, for $0 \leq \theta \leq 360^\circ$, giving your answers to 1 decimal place. (3 marks)

a) $R \sin \theta \cos \alpha + R \cos \theta \sin \alpha = 2 \sin \theta + 4 \cos \theta$

$$\frac{R \sin \alpha = 4}{R \cos \alpha = 2}$$

$$\tan \alpha = 2 \quad \alpha = 63.43^\circ (2dp)$$

$$R^2 = 4^2 + 2^2 \Rightarrow R = \sqrt{20} = 2\sqrt{5}$$

$$2\sqrt{5} \sin(\theta + 63.43^\circ)$$

b) $2\sqrt{5} \sin(\theta + 63.43^\circ) = 3$

$$\sin(\theta + 63.43^\circ) = \frac{3}{2\sqrt{5}}$$

$$\theta + 63.43^\circ = \cancel{42.13}, 180 - 42.13, 360 + 42.13$$

$$\theta = 74.44^\circ, \dots$$