

In questions that tell you to show your working, you shouldn't depend solely on a calculator. For these questions, solutions based entirely on graphical or numerical methods are not acceptable.

- 1 a Simplify these expressions.

i $2^m \times 2^n$ ii $\frac{5^{m+1}}{5^{2n}}$ iii $(3^m)^2 \times \sqrt{(3^m)}$

[5 marks]

- b Given $\frac{16^p \times 8^q}{4^{p+q}} = 2^n$, write down an expression for n in terms of p and q

[3]

- 2 a Simplify these surds. You must show your working.

i $\frac{12}{\sqrt{3}}$ ii $(3+2\sqrt{2})(5-4\sqrt{2})$ iii $\frac{3-\sqrt{7}}{1+3\sqrt{7}}$

[8]

- b A rectangle $ABCD$ has an area of 8 cm^2 and length $(3-\sqrt{5}) \text{ cm}$.

Work out its width, giving your answer as a surd in simplified form. Show your working.

[3]

- 3 What is the equation of the straight line that is perpendicular to $3x + 2y = 5$ and that passes through the point $(4, 5)$?

[5]

- 4 a Express $x^2 + 6x + 13$ in the form $(x + a)^2 + b$

[2]

- b Hence sketch the curve $y = x^2 + 6x + 13$ and label the vertex, and the point where the curve cuts the y -axis.

[3]

- 5 Solve these simultaneous equations. You must show your working.

$2x + y = 3$ $3x^2 + 2xy + 7 = 0$

[8]

- 6 Prove that the equation $x = 1 + \frac{2x-5}{x+4}$ has no real solutions.

[4]

- 7 a Solve these inequalities. You must show your working.

i $3x - 5 < 11 - x$ ii $x^2 - 6x + 5 \leq 0$

[5]

- b Show on a graph the set of values of x that satisfy both $3x - 5 < 11 - x$ and $x^2 - 6x + 5 \leq 0$

[2]

- 8 PQR is a right-angled triangle. Write an exact expression for x , show your working.

[6]

- 9 The equation of a circle is $x^2 + y^2 - 10x + 2y - 23 = 0$

- a Showing your working clearly, work out

i Its centre, ii Its radius.

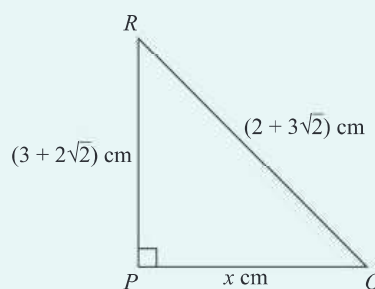
- b The line $y = x + 2$ meets the circle at the points P and Q

Work out, in exact form, the coordinates of P and Q . Show your working.

[5]

- 10 The quadratic equation $(k+1)x^2 - 4kx + 9 = 0$ has distinct real roots. What range of values can k take?

[6]



11 Prove that $\frac{a+b}{2} \geq \sqrt{ab}$ for all positive numbers a and b [4]

12 a Factorise the expression $2u^2 - 17u + 8$ [2]

b Hence solve the equation $2^{2x+1} - 17 \times 2^x + 8 = 0$. Show your working. [3]

13 The straight line $y = mx + 2$ meets the circle $x^2 + y^2 + 4x - 6y + 10 = 0$

a Prove that the x -values of the points of intersection satisfy the equation $(m^2 + 1)x^2 + 2(2 - m)x + 2 = 0$ [4]

b The straight line $y = mx + 2$ is a tangent to the circle $x^2 + y^2 + 4x - 6y + 10 = 0$. What are the possible values of m ? Give your answers in exact form. [5]

14 a Given $9^2 = 3^n$, write down the value of n [1]

b Solve these simultaneous equations. Show your working.

$$3^{x+y} = 9^2 \quad 4^{x-2y} = 8^4 \quad [4]$$

15 Decide which of these statements are true and which are false.

For those that are true, prove that they are true.

For those that are false, give a counter-example to show that they are false.

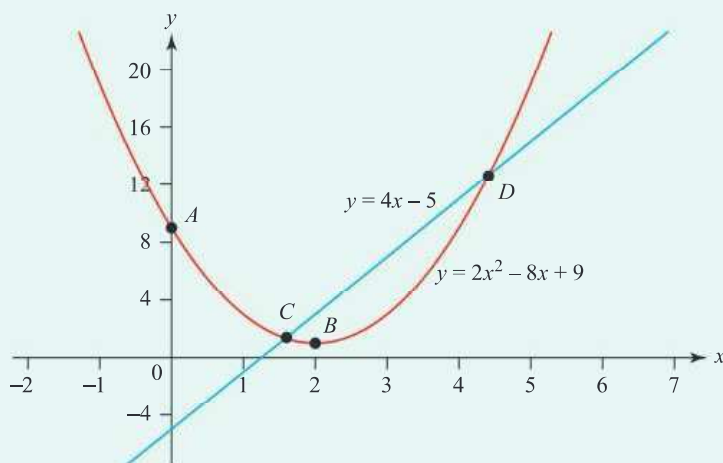
a If $a > b$ then $a^2 > b^2$ [2]

b $n^2 + n$ is an even number for all positive integers n [3]

c If a and b are real numbers then $b^2 \geq 4a(b - a)$ [3]

d $2^n - 1$ is prime for all positive integers n [2]

16 The diagram shows the parabola $y = 2x^2 - 8x + 9$ and the straight line $y = 4x - 5$



Showing your working clearly, work out the coordinates of the following points.

a A, the y -intercept of the parabola. [1]

b B, the vertex of the parabola. [2]

c C and D, the points of intersection of the line and parabola. [4]

17 Prove that the circle $x^2 + y^2 + 6x - 4y - 2 = 0$ lies completely inside the circle $x^2 + y^2 - 2x - 10y - 55 = 0$ [9]