

CHAPTER 8

8.1 Pascal's triangle

- 1 **a** $243 - 405x + 270x^2 - 90x^3 + 15x^4 - x^5$
 b $y^4 + 8xy^3 + 24x^2y^2 + 32x^3y + 16x^4$
 c $243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$
- 2 **a** 160 **b** -1080 **c** $-\frac{3}{2}$
- 3 $1 + 7x + 9x^2 - 27x^3 - 54x^4$
- 4 **a** $16 + 96x + 216x^2 + 216x^3 + 81x^4$
 b $16 - 96x + 216x^2 - 216x^3 + 81x^4$
- 5 $k = \pm 5$
- 6 **a** $64 + 48k + 12k^2 + k^3$
 b $64 - 48x + 60x^2 - 25x^3 + 15x^4 - 3x^5 + x^6$
- 7 $k = 1$ or $k = -\frac{3}{5}$

8.2 Factorial notation

- 1 **a** 120 **b** 1320 **c** 380
- 2 **a** 11 440 **b** 792
 c 646 646 **d** 31 824
- 3 $p = {}^4C_2, q = {}^5C_3, r = {}^6C_3, s = {}^6C_4$
- 4 ${}^{15}C_7$
- 5 **a** 0.162 (3 s.f.) **b** 0.0130 (3 s.f.)
- 6 $p = 14$
- 7 $q = 22$

- 8 **a** ${}^{k+2}C_k = \frac{(k+2)!}{k!(k+2-k)!} = \frac{(k+1)(k+2)}{2}$
 b $k = 7$ **c** k is a positive integer > 0

8.3 The binomial expansion

- 1 **a** $1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$
 b $64 + 192x + 240x^2 + 160x^3 + 60x^4 + 12x^5 + x^6$
 c $10\,000 + 4000x + 600x^2 + 40x^3 + x^4$
- 2 **a** $81 - 216x + 216x^2 - 96x^3 + 16x^4$
 b $1024 + 640x + 160x^2 + 20x^3 + \frac{5x^4}{4} + \frac{x^5}{32}$
 c $64 - 192x + 240x^2 - 160x^3 + 60x^4 - 12x^5 + x^6$
- 3 **a** $1 + 10x + 45x^2 + 120x^3$
 b $1 + 15x + 105x^2 + 455x^3$
 c $512 + 2304x + 4608x^2 + 5376x^3$
- 4 **a** $1 - 5x + \frac{45x^4}{4} - 15x^3$
 b $512 - 768x + 512x^2 - \frac{1792x^3}{9}$
 c $6561 + 34\,992x + 81\,648x^2 + 108\,864x^3$
- 5 $32 - 240x + 720x^2 - 1080x^3$
- 6 $A = 531\,441, B = -2\,125\,764, C = 3\,897\,234$
- 7 $1 - 6x + 16x^2 - \frac{224x^3}{9}$
- 8 $x^5 - 10x^3 + 40x - \frac{80}{x} + \frac{80}{x^3} - \frac{32}{x^5}$

8.4 Solving binomial problems

- 1 **a** 364 **b** 15 360 **c** -21 875
- 2 **a** 489 888 **b** 354 375 **c** 7920
- 3 10 800
- 4 $\frac{160}{27}$
- 5 $n = 6$
- 6 $a = \pm 4$
- 7 0.5
- 8 **a** $256 - 3072x + 16\,128x^2 - 48\,384x^3$ **b** -40 320
- 9 **a** $729 + 1458px + 1215p^2x^2$ **b** $p = 6$

8.5 Binomial estimation

- 1 **a** $1 - \frac{9x}{5} + \frac{36x^2}{25} - \frac{84x^3}{125}$ **b** 0.8728
- 2 **a** $1 - 12x + 60x^2$
 b Expand $(3 + x)(1 - 2x)^6$ up to terms in x^2
- 3 **a** $6561 + 4374x + \frac{5103x^2}{4} + \frac{1701x^3}{8}$
 b Using $x = 0.2$, 7488.531
- 4 **a** $1 + \frac{9x}{4} + \frac{9x^2}{4} + \frac{21x^3}{16}$
 b Using $x = 0.06$, $(1.015)^9 = 1.143$ (3 d.p.)
- 5 **a** $1 - 18x + 135x^2 - 540x^3$
 b Expand $\left(1 + \frac{x}{2}\right)(1 - 3x)^6$ up to terms in x^2
- 6 **a** $128 - 224x + 168x^2$
 b Substitute $x = 0.03$ in the expansion
- 7 **a** $59\,049 - 39\,366x + \frac{59\,049x^2}{5} - \frac{52\,488x^3}{25}$
 b 58 072.198 320 **c** $1.64 \times 10^{-7}\%$ (3 s.f.)

Problem solving: Set A

- B** **a** $k = 2$ **b** 2160
- S** **a** $2048 - 11\,264px + 28\,160p^2x^2$
 b $p = -4, q = 45\,056$
- G** $k = 2$

Problem solving: Set B

- B** **a** $1 - 20x + 150x^2$ **b** $A = 2$
 c $k = 3$ **d** $B = 240$
- S** $A = 16, k = 6, B = -360$
- G** **a** $p = 12, q = \frac{1}{3}$ **b** $k = \frac{130}{9}$