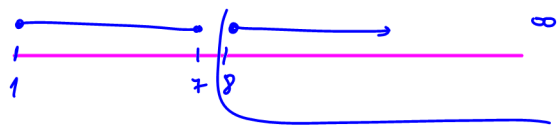


11.10

$$\sum_{r=0}^{\infty} 3 \times 0.8^r = S_{\infty} - S_7$$



$$r=1 \quad u_1 = 3 \times 0.8 = 2.4$$

$$r=2 \quad u_2 = 3 \times 0.8^2 = 1.92$$

$$S_{\infty} = \frac{a}{1-r} = \frac{2.4}{1-0.8} = 12$$

$$S_7 = a \frac{1-r^7}{1-r} = 2.4 \frac{1-0.8^7}{1-0.8} = 9.48$$

- 2
- A function is defined by $f(x) = x^2 - 2, x \in \mathbb{R}, x \geq 0$
- a

State the range of $f(x)$

[1]
- b

Write an expression for the inverse function $f^{-1}(x)$, stating its domain.

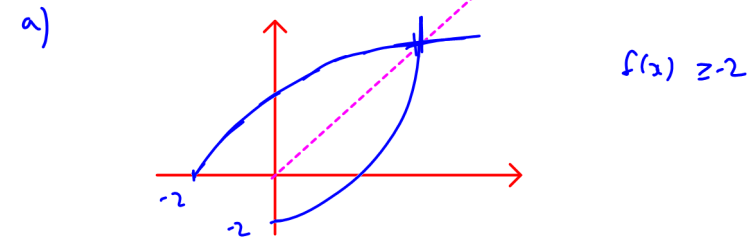
[3]
- c

Sketch the graphs of $f(x)$ and $f^{-1}(x)$ on the same set of axes.

[4]
- d

Find the value of x for which $f(x) = f^{-1}(x)$

[3]



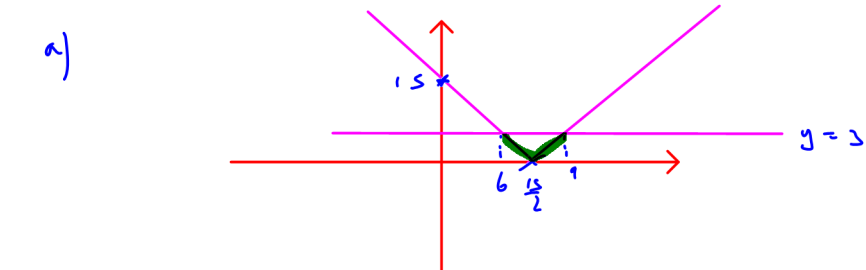
b) $y = x^2 - 2 \Rightarrow f^{-1}(x) = \sqrt{y+2}$

- 4
- a

Sketch the graph of $y = |2x - 15|$
- b

Solve the equation $|2x - 15| = 3$
- c

Solve the inequality $|2x - 15| \leq 3$

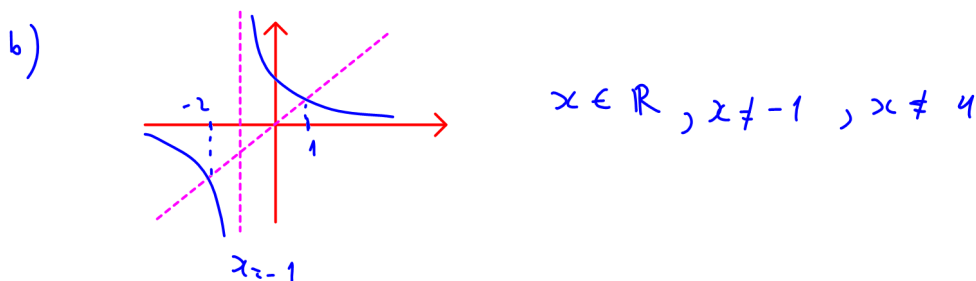


b) +ve $2x - 15 = 3 \Rightarrow x = 9$
-ve $-(2x - 15) = 3 \Rightarrow -2x + 15 = 3 \Rightarrow x = 6$

c) $6 \leq x \leq 9$

- 7 The function $f(x)$ is defined by $f(x) = \frac{3x-7}{x^2-3x-4} - \frac{1}{x-4}$
- a Show that $f(x) = \frac{2}{x+1}$ [4]
- b What is the largest possible domain of $f(x)$? $x=4$ [1]
- c Work out an expression for the inverse function $f^{-1}(x)$, stating its domain. [2]
- d Solve the equation $f(x) = f^{-1}(x)$. Show your working. [3]

$$a) \quad \frac{3x-7}{(x-4)(x+1)} - \frac{1}{(x-4)(x+1)} = \dots = \frac{2}{x+1}$$



$$c) \quad y = \frac{2}{x+1} \quad f^{-1}(x) = \frac{2-x}{x}$$

$$d) \quad \frac{2}{x+1} = x \Rightarrow \begin{aligned} 2 &= x^2 + x \\ x^2 + x - 2 &= 0 \\ (x+2)(x-1) &= 0 \\ x &= -2 \quad \text{or} \quad x = 1 \end{aligned}$$

- 12 If $\frac{6x^4+5x^3-4x^2-3x+1}{2x+3} \equiv Ax^3+Bx^2+Cx+D+\frac{E}{2x+3}$, find the constants A, B, C, D and E [6]

$$\begin{array}{r} 3x^4 - 2x^2 + x - 3 \\ 2x+3 \overline{) 6x^4 + 5x^3 - 4x^2 - 3x + 1} \\ \underline{6x^4 + 9x^3} \\ -4x^3 - 4x^2 \\ \underline{-4x^3 - 6x^2} \\ 2x^2 - 3x \\ \underline{2x^2 + 3x} \\ -6x + 1 \\ \underline{-6x - 9} \\ 10 \end{array}$$

$3x^3 - 2x^2 + x - 3 \equiv \frac{10}{2x+3}$

13 Functions $f(x)$ and $g(x)$ are defined by

$$f(x) = e^{2x}, x \in \mathbb{R}, \quad \text{and} \quad g(x) = \ln(3x-2), x \in \mathbb{R}, x > \frac{2}{3}$$

- a** Write an expression for $fg(x)$ [3]
b Solve the equation $fg(x) = x^2$. Show your working. [4]
c Work out an expression for $f^{-1}(x)$ [2]
d Solve the equation $f(x) = 5$. Show your working. [2]

a) $f(g(x)) = e^{2g(x)}$
 $= e^{2\ln(3x-2)}$
 $= (3x-2)^2$

b) $(3x-2)^2 = x^2$
 $9x^2 - 12x + 4 = x^2$
 $8x^2 - 12x + 4 = 0 \Rightarrow \pm 4$ solvable

c) $y = e^{2x} \Rightarrow x = e^{2y}$
 $y = \frac{\ln x}{2} = \ln x^{1/2}$

4 A car costs £30 000. Its value depreciates by 20% per annum. Work out

- a** Its value after 1 year, $\rightarrow h=2$ [1]
b Its value after 4 years, $\rightarrow n=5$ [2]
c The year in which it will be worth less than £5000 [4]

a) $a = 30,000$ $r = 0.8$

$V = 30,000 \times 0.8^{n-1}$
 $V = 30,000 \times 0.8^n$

$n=1$ present
 $n=0$ present

$V = 30,000 \times 0.8^{1-1} = 24,000$

c) $30,000 \times 0.8^{n-1} < 5,000$ $n-1 < \log_{0.8}(1/6)$
 $n < \dots$

8 When $(1+ax)^n$ is expanded the coefficients of x and x^2 are -4 and 20 respectively.

- a** Work out the value of a and the value of n [8]
b Evaluate the coefficient of x^3 [2]

a) $(1+ax)^n = 1 + na x + \frac{n(n-1)}{2!} a^2 x^2$

$an = -4$, $\frac{a^2 n(n-1)}{2} = 20$

$a^2 n^2 - a^2 n = 40 \Rightarrow (an)^2 - an(n) = 20$
 $(-4)^2 - (-4)a = 20$
 $a, n = \dots$

10 a Use a formula to evaluate $\sum_{r=1}^{40} (3r+1)$

b Calculate the value of n for which $\sum_{r=1}^n (3r+1) = 9800$

$$b) \quad a = 1, \quad d = 3 \quad S_n = 9800$$

$$\frac{n}{2} (2 \times 1 + (n-1) 3) = 9800 \quad \text{formula}$$

13 The first three terms of a geometric series are $(3p-1)$, $(p-3)$ and $(2p)$ respectively.

a Use algebra to work out the possible values of p [5]

b For the negative value of p , calculate the sum to infinity of the series. [3]

c For the positive value of p , evaluate the sum of the first 999 terms of the series. [2]

$$a) \quad \frac{p-3}{3p-1} = \frac{2p}{p-3} \Rightarrow p^2 - 6p + 9 = 6p - 2p$$

$$S_p^2 + 4p - 9 = 0 \quad p = 1, \quad p = -9/5$$

$$b) \quad p = -9/5 \quad \begin{matrix} \nearrow 3/4 \\ -32/5, -24/5, -18/5 \end{matrix}$$

$$a = -\frac{32}{5} \quad r = 3/4 \quad S_{\infty} = \frac{-\frac{32}{5}}{1-3/4}$$

$$c) \quad \begin{matrix} \xrightarrow{x-1} \\ 2, -2, 2 \end{matrix}$$

$$\Sigma 999 = 999 \times (1-2) + 2 = \textcircled{2}$$

$$S_n = a \frac{1-r^n}{1-r} = 2 \left(\frac{1-(-1)^{999}}{1-(-1)} \right) \stackrel{or}{=}$$

$$= 2 \left(\frac{1+1}{1+1} \right) = \textcircled{2}$$