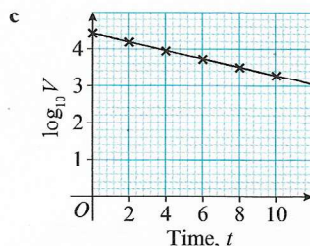


4 a

Time, t	0	2	4	6	8	10
$\log_{10} V$	4.40	4.17	3.94	3.72	3.49	3.26

b $V = ab^t$
 $\log V = \log(ab^t)$
 $\log V = \log a + \log b^t$
 $\log V = \log a + t \log b$

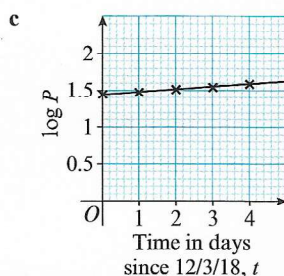


d $\frac{3.26 - 4.4}{10 - 0} = -0.114$ e $a = 25\,100$, $b = 0.769$

5 a

Time in days since 12/3/18, t	0	1	2	3	4
$\log P$	1.44	1.47	1.51	1.54	1.58

b $P = ab^t$
 $\log P = \log(ab^t)$
 $\log P = \log a + \log b^t$
 $\log P = \log a + t \log b$



d $a = 27.43$, $b = 1.084$

- 6 a $\log_4 P = -0.075t + 5.2$ b 1350
c $a = 1350$, $b = 0.901$ d 59 people
7 a $\log_2 N = 0.26t + 9.3$ b $a = 630.3$, $b = 1.2$
c The initial number of bacteria in the petri dish.

Problem solving: Set A

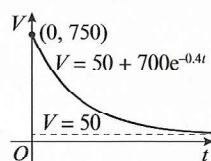
B 1.25

S a $9^x - 3^{x+2} + 20 = 0$ b $\log_3 4$, $\log_3 5$
 $(3^2)^x - (3^x)(3^2) + 20 = 0$
 $(3^x)^2 - 9(3^x) + 20 = 0$
 $u^2 - 9u + 20 = 0$

G $\frac{1}{2} \cdot \frac{\log 6}{2 \log 2}$

Problem solving: Set B

B a $V = 50 + 700e^{-0.4t}$ b £144.73

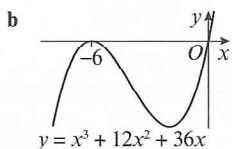


c $-\frac{5}{2} \ln \frac{1}{14}$ or $\frac{5}{2} \ln 14$

- S a 14643 computers b $\frac{dC}{dt} = \frac{1}{4}e^{\frac{1}{4}t}$
c $4 \ln 48$
G a Temperature of soup will decrease, so k must be negative.
b $a = 45$ c $\frac{1}{5} \ln \frac{2}{3}$ d 10.96 minutes

Exam Question Bank

- 1 a 243 b $8x^{-\frac{1}{2}}$
2 $\frac{13}{23}\sqrt{3} - \frac{1}{23}$
3
4 a $2\sqrt{2}$ b $4\sqrt{6}$
5 a $\frac{2\sqrt{5}}{5}i - \frac{\sqrt{5}}{5}j$ b 26.6° (1 d.p.) below the x -axis
6 $\frac{27}{5}x^{\frac{5}{3}} - \frac{4}{x} - \frac{3}{4}x^4 + 2x + c$
7 LHS $= \frac{2\sin^2 x - (\sin^2 x + \cos^2 x)}{\cos^2 x} = \frac{\sin^2 x - \cos^2 x}{\cos^2 x}$
 $= \frac{\sin^2 x}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x} = \tan^2 x - 1 \equiv \text{RHS}$
8 $\frac{203}{64}$
9 $n = 5$
10 a $A(5 - 2\sqrt{3}, 15 - 4\sqrt{3})$, $B(5 + 2\sqrt{3}, 15 + 4\sqrt{3})$
b $x^2 - 8x + 18 \leq y \leq 2x + 5$
11 $x = \frac{1}{16}$, $x = \frac{81}{16}$
12 a i 18 ii $c = \frac{3}{2}$
b $f(x) = 4x^3 - 4x^2 - 15x + 18 \Rightarrow f'(x) = 12x^2 - 8x - 15$
13 a $k^2 - 10k + 4$ b $k = 5 \pm \sqrt{21}$
c Substituting $k = 8$ into $f(x)$ gives
 $f(x) = x^2 + 4x + 7 = (x + 2)^2 + 3$
So $f(x) > 0$ for all values of x
14 a $p = \frac{5}{3}$ b $\frac{38}{3}$
15 a $f'(x) = \frac{1}{6\sqrt{x}} - \frac{3}{2\sqrt{x^3}}$ b (9, 0)
c $f''(x) = -\frac{1}{12}x^{-\frac{3}{2}} + \frac{9}{4}x^{-\frac{5}{2}}$
 $f''(9) = \frac{1}{162}$ therefore local minimum
16 a Ship: $-2i + 6j$, boat: $12i - 4j$ b $2\sqrt{74}$
17 a e.g. $k = -15^\circ$, $k = 165^\circ$
b e.g. $x = 105^\circ$, $x = -75^\circ$
18 $-\frac{7}{3}$
19 $8i - 6j$
20 a $1 + \frac{4}{x} + \frac{4}{x^2}$ b $1 + \frac{12x}{5} + \frac{12x^2}{5} + \frac{32x^3}{25}$
c $\frac{428}{25}$
21 a $2x + 5y - 66 = 0$ b 208.8
22 a $x = \frac{3}{2}$, $x \leq -1$ b $x = \frac{3}{4}$, $x = -\frac{1}{2}$
23 $25k^2 - 20k < 0 \Rightarrow 5k(5k - 4) < 0 \Rightarrow 0 < k < \frac{4}{5}$
24 a $x(x + 6)(x + 6)$

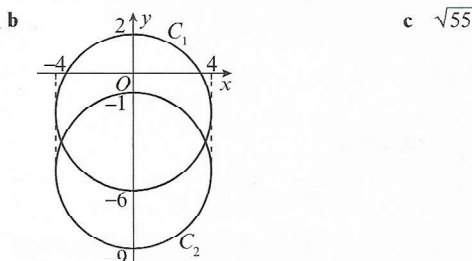


25 $\frac{1}{3} \ln \frac{4}{3}, \frac{1}{3} \ln 5$

26 25

27 $a = \frac{8}{9}, b = -\frac{5}{3}, c = 1$

28 a C_1 : radius 4, centre (0, -2); C_2 : radius 4, centre (0, -5)



29 a $q^{100} + 100q^{99}p + 4950q^{98}p^2 + 161700q^{97}p^3$

b 0.85896

30 a Consider $(p - q)^2 = p^2 + q^2 - 2pq = p^2 - q^2 + 2q^2 - 2pq$
If $p > q > 0$ then $2pq > 2q^2$, so $2q^2 - 2pq$ is negative, and
 $(p - q)^2 < p^2 - q^2$, so $\sqrt{p^2 - q^2} > p - q$ as required.

b $p = 0, q = 0$ would give $0 > 0$ which is untrue.

31 $-\frac{8}{\sqrt{17}}$

32 a $x > 1$ b $-3 \leq x \leq 7$ c $1 < x \leq 7$

33 a (5, -4.5) b 10

c $x \geq 5$ d $k \leq 1.5$

34 a When $x = -1$, $5^x = 5^{-1} = \frac{1}{5}$, $2^x = 2^{-1} = \frac{1}{2}$ and $\frac{1}{5} < \frac{1}{2}$, so not true for all values of x .

b Write the consecutive odd numbers as $2n - 1$ and $2n + 1$, where n is an integer. Then:

$$\begin{aligned} \frac{1}{2}(2n - 1)^2 + \frac{1}{2}(2n + 1)^2 &= \frac{1}{2}(4n^2 - 4n + 1) + \frac{1}{2}(4n^2 + 4n + 1) \\ &= 2n^2 - 2n + \frac{1}{2} + 2n^2 + 2n + \frac{1}{2} \\ &= 4n^2 + 1 \end{aligned}$$

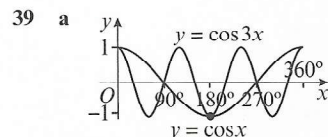
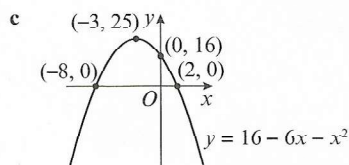
which is one more than an even number, so must be odd.

35 $x = 0, y = 2$ or $x = 6, y = -4$

36 $x = \frac{3}{2}$ or $x = \frac{5}{2}$

37 a 34.8° and 145.2° (1 d.p.) b 19.2 cm (3 s.f.)

38 a $p = 3, q = 25$ b 100



b Stretch with scale factor $\frac{1}{3}$ parallel to the x-axis.

c $x = 20^\circ, 100^\circ, 140^\circ, 220^\circ, 260^\circ$ or 340°

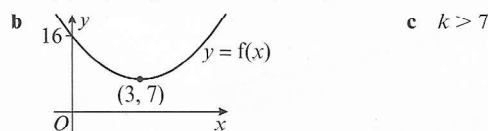
40 a $1024 - \frac{1024}{3}x + \frac{256}{5}x^2$ b $p = \frac{1}{2}$ c $q = \frac{1}{12}$

41 a $(x + 1)(2x + 1)(3x - 2)$

b $\frac{6x^3 + 5x^2 - 3x - 2}{2x^3 + 3x^2 + x} = \frac{(x + 1)(2x + 1)(3x - 2)}{x(2x + 1)(x + 1)} = 3 - \frac{2}{x}$

42 a $(x - 2)^2 + (y + 1)^2 = 72$ b $8\sqrt{17} - 4$

43 a $f(x) = (x - 3)^2 - 9 + 16 = (x - 3)^2 + 7$



44 a $A(-1, 1), B(2, 4)$

b $\frac{9}{2}$

45 15.4 cm² (3 s.f.)

46 a $y = x - 0.05x^2$ b 20 m

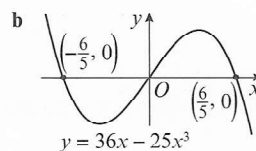


c Model gives maximum height of 5 m

47 $x = 68.2^\circ, 146.3^\circ, 248.2^\circ$ or 326.3°

48 a $3x^2 + 6x - 9$ b $\{x : x < -3\} \cup \{x : x > 1\}$

49 a $x(6 + 5x)(6 - 5x)$



c $A(-1, -11)$ and $B(1, 11)$

Distance is $\sqrt{(-1 - 1)^2 + (-11 - 11)^2} = 2\sqrt{122}$

50 a (1, 2.5) b $y = -\frac{2}{7}x + \frac{64}{7}$

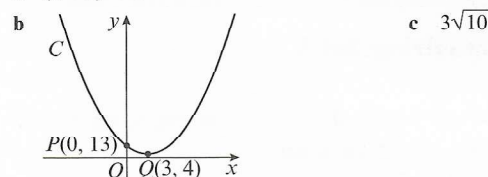
c 128

51 a $1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$

b i $(1 + \sqrt{5})^5 = 1 + 5\sqrt{5} + 10(5) + 10(5\sqrt{5}) + 5(25) + (25\sqrt{5})$
 $= 176 + 80\sqrt{5}$

ii $\log_2(1 + \sqrt{5})^5 = \log_2(176 + 80\sqrt{5}) = \log_2 16(11 + 5\sqrt{5})$
 $= \log_2 16 + \log_2(11 + 5\sqrt{5}) = 4 + \log_2(11 + 5\sqrt{5})$

52 a $(x - 3)^2 + 4$



53 a 4

b $\frac{208}{15}$

54 a $\log_3 \frac{7x - 18}{(x - 2)^2} = 1$

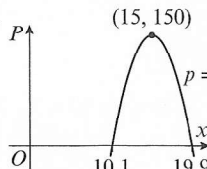
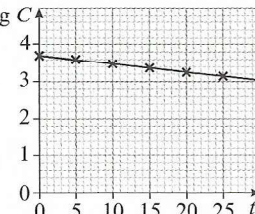
$7x - 18 = 3x^2 - 12x + 12$

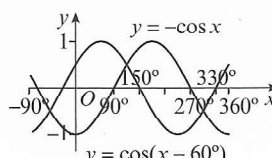
$3x^2 - 19x + 30 = 0$

b $x = 3, x = \frac{10}{3}$

55 a $2x - 4 = x^2 + px - 2p \Rightarrow x^2 + (p - 2)x + 4 - 2p = 0$
So $(p - 2)^2 - 4(4 - 2p) < 0 \Rightarrow p^2 + 4p - 12 < 0$

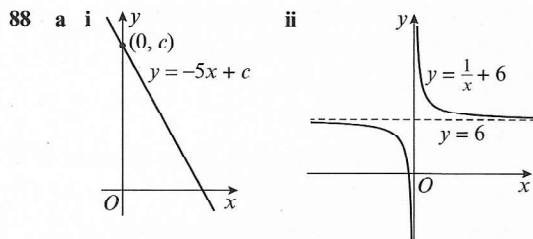
b $-6 < p < 2$

- 56 a $|DE| = \sqrt{58}$, $|EF| = \sqrt{89}$ and $|DF| = \sqrt{65}$, so triangle DEF is scalene.
b 50.9°
- 57 a Factor theorem states if $(x+3)$ is a factor of $g(x)$ then $g(-3) = 0$.
 $g(-3) = 4 \times (-3)^3 - 16 \times (-3)^2 - 35 \times (-3) + 147$
 $= -108 - 144 + 105 + 147 = 0$
 so $(x+3)$ is a factor of $g(x)$.
 b By division $g(x) = (x+3)(4x^2 - 28x + 49)$
 $= (x+3)(2x-7)^2$
 c i $g(x) \leq 0$ when $x \leq -3$
 ii $g(2x) \leq 0$ when $x \leq -\frac{3}{2}$
- 58 a i $(4, -6)$ ii 6 b $0 < k < 2.4$
- 59 a £805.19 b $24.5e^{0.035t}$ c 19.8 years
- 60 a Use $\sin^2 x = 1 - \cos^2 x$ and rearrange
b $x = 36.42^\circ, 97.94^\circ, 262.06^\circ$ or 323.58°
- 61 a $1 + 12x + 54x^2 + 108x^3 + 81x^4$
b Expand $(1-3x)^4$ in a similar way to $(1+3x)^4$ and add the two expansions. The x and x^3 terms cancel out.
c $(0, 2)$
- 62 $\frac{1}{2}$ and -2
- 63 a Using cosine rule, $\frac{3}{4} = \frac{6^2 + (2x-5)^2 - (x+2)^2}{2 \times 6 \times (2x-5)}$
b $x = 7 \pm \sqrt{15}$
- 64 a $1 + x^2 < 1 + x^2 + 2x \Rightarrow 0 < 2x \Rightarrow x > 0$
b For $x = -1$, $1 + (-1)^2 < (1 + (-1))^2 \Rightarrow 2 < 0$ (not true)
- 65 $k = -11$
- 66 a $x = 45^\circ$ or 105°
b $x = 67.5^\circ, 112.5^\circ, 247.5^\circ$ or 292.5°
- 67 a i $P = 150 - 6.25(x-15)^2$ ii 10.1 and 19.9
 iii 
 b The company would make an annual loss of £475 000.
 c i £150 000 ii £15
- 68 a $C = ab^t$
 $\log C = \log(ab^t)$
 $\log C = \log a + \log b^t$
 $\log C = \log a + t \log b$
 b  c $a = 5100$, $b = 0.95$
- 69 a 7
b $f(1) = (1+k)(2+5) + 7 = 0$, $7 + 7k + 7 = 0$ so $7k = -14$ and $k = -2$
c $x = -\frac{1}{2}$, $x = -3$ and $x = 1$

- 70 a i $(5, -3)$ ii $3\sqrt{5}$
b $(x-5)^2 + y^2 = 36$
- 71 a $x^2 - 8x + 17 = (x-4)^2 + 1$. Minimum value is $y = 1$.
b Sometimes true; e.g. substitute in -5 and 5 to obtain different results.
- 72 a $(-3, 0), (5, 0)$ b $\frac{863}{6}$
- 73 a 258.6 m (1 d.p.) b 087° (nearest degree)
- 74 a Use $P = 300 = 2\pi r + 2y$ to find $y = 150 - \frac{\pi}{2}x$
 Substitute into $A = \frac{\pi x^2}{4} + xy$
 b $\frac{22500}{\pi} \text{ m}^2$
- 75 a $f(5) = 250 + 25k - 120 + 45 = 0$ so $25k = -175$ and $k = -7$
 b $f(x) = (x-5)(2x-3)(x+3)$
 c $5^y = 5$ so $y = 1$. $5^y = \frac{3}{2} \Rightarrow y \ln 5 = \ln \frac{3}{2}$ so $y = 0.25$
 ($5^y = -3$ has no solutions)
- 76 $x = 49.0^\circ, 84.4^\circ$ or 169.0°
- 77 a $y = 3x$ b $(\frac{6}{5}, \frac{18}{5})$
 c $\sqrt{(\frac{6}{5})^2 + (\frac{18}{5})^2} = \sqrt{\frac{360}{25}} = \frac{6\sqrt{10}}{5}$ d $2\sqrt{10}$
 e 12
- 78 a 
 b $(0, 0.5), (-30^\circ, 0), (150^\circ, 0), (330^\circ, 0)$
 c $x = -60^\circ, 120^\circ$ or 300°
- 79 a $(\sqrt[3]{16}, 8)$
 b $4x^{\frac{3}{2}} - \frac{1}{2}x^3 = 0 \Rightarrow x^3 - 8x^{\frac{3}{2}} = 0$
 $\Rightarrow x^{\frac{3}{2}}(x^{\frac{3}{2}} - 8) = 0 \Rightarrow x^{\frac{3}{2}} = 0$ or $x^{\frac{3}{2}} = 8$
 $\Rightarrow x = 0$ or $x = 4$, so B is the point $(4, 0)$.
 c $\frac{96}{5}$
- 80 a $\frac{3}{5}\mathbf{a} - \frac{2}{5}\mathbf{b}$
 b $\overrightarrow{CA} = \mathbf{a} - \mathbf{b}$. As $\overrightarrow{CA}$ and $\overrightarrow{CZ}$ are not parallel, $\overrightarrow{CZ}$ will not pass through the point A .
 c Create a vector equation using $\overrightarrow{OZ} = \overrightarrow{OA} + \overrightarrow{AP} + \overrightarrow{PZ}$ and equate $\mathbf{a}$ and $\mathbf{b}$ parts.
- 81 a $(2, 9)$ b 40 c $3 < k < 15$
- 82 a $(x-5)^2 + (y-3)^2 = 100$
b $4x + 3y - 79 = 0$ c $10\sqrt{5}$
- 83 a $f(-2) = 0$
 b $f(x) = (x+2)(x^2 - 4x + 9)$; using discriminant on quadratic, $b^2 - 4ac = 16 - 36 = -20 < 0$ so no real roots. Only one root when $x+2 = 0$, i.e. $x = -2$
 c $x^3 - 2x^2 + x + 18 \geq 18$, means $x^3 - 2x^2 + x \geq 0$. Factorising $x(x^2 - 2x + 1) = x(x-1)^2$, $(x-1)^2 \geq 0$ for all values of x . If $x \geq 0$ then $x(x-1)^2 \geq 0$ and $f(x) = x^3 - 2x^2 + x + 18 = x(x-1)^2 + 18 \geq 18$ for all $x \geq 0$.
- 84 a $f(x) = (x+2)(2x-1)(3x+1)$
b $y = 30^\circ, 150^\circ, 199.5^\circ$ or 340.5°
- 85 a $\theta = 33.4^\circ, 176.6^\circ, 213.4^\circ$ or 356.6°
b $x = 379.47^\circ$ or 520.53°

86 a $4x + 3y - 13 = 0$ b $3x - 4y + 34 = 0$
c -10 d $(x + 3)^2 + y^2 = 50$

87 a $3x + 4y = 19$
b $3 \times 13 + 4y = 19 \Rightarrow y = -5$
c $(x - 13)^2 + (y + 5)^2 = 125$
d $50\sqrt{5}$



b $\frac{1}{x} + 6 = -5x + c$ rearranges to $5x^2 + (6 - c)x + 1 = 0$
Using $b^2 - 4ac > 0$, $(6 - c)^2 - 20 > 0 \Rightarrow (6 - c)^2 > 20$
c $c < 6 - 2\sqrt{5}$ or $c > 6 + 2\sqrt{5}$

89 a 4 and 8 b (7, 5)

90 a Multiply dimensions: x , $36 - 2x$, $60 - 2x$
b $16 - 2\sqrt{19}$ c 7092.4 cm^3

91 a Perimeter $= 500 = 2\pi r + 2x \Rightarrow x = 250 - \pi r$
Area $= \pi r^2 + 2rx = \pi r^2 + 2r(250 - \pi r) = 500r - \pi r^2$

b $A = \frac{62500}{\pi}$, $r = \frac{250}{\pi}$

92 a -2 b $(x - 3)^2 + (y + 2)^2 = 169$
c 78

93 a Gradient of $PQ = \frac{15}{25} = \frac{3}{5}$; gradient of $SR = \frac{9}{15} = \frac{3}{5}$
So PQ and SR are parallel.

b Length of $PS = \sqrt{194}$; length of $QR = \sqrt{74}$
So trapezium is not isosceles.

c $(\frac{5}{2}, \frac{31}{4})$

d Midpoint of PR is $(0, 5)$; midpoint of SQ is $(5, 8)$, so neither diagonal of the trapezium bisects the other.