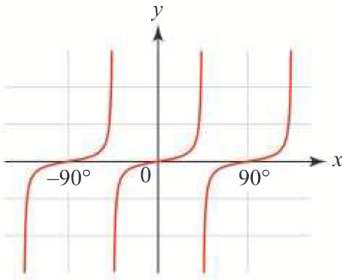


Assessment 14

1 a $x = 60^\circ, -120^\circ$

b



$x = \pm 45$

2 a $\frac{25\pi}{3}$

b $\frac{10\pi}{3}$

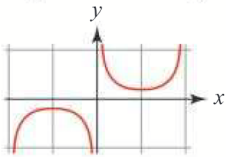
3 a 1.2 rad

b 9.6 cm

4 a i $\sqrt{2}$

ii $-\sqrt{3}$

b



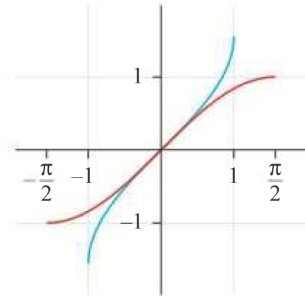
c $x = \pi, x = -\pi, x = 0$

$y \in \mathbb{R}, y > 1, y < -1$

5 a $x = 30^\circ, 150^\circ$

b $x = 135^\circ, 225^\circ$

6 a



b Reflection in line $y = x$

c i $-\frac{\pi}{3}$

ii $\frac{\pi}{6}$

7 a 5 ms^{-1} b $0, \frac{2\pi}{3}(2.09) \text{ s}, \frac{14\pi}{3}(14.66) \text{ s}, \frac{26\pi}{3}(27.2) \text{ s}$

8 $x = \frac{\sqrt{2}}{2} - \frac{1}{2}$

9 a $\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} \equiv \frac{1}{\cos^2 \theta}$
 $\tan^2 \theta + 1 \equiv \sec^2 \theta$
 $\sec^2 \theta - \tan^2 \theta \equiv 1$

b i $\sec \theta = \pm \sqrt{6}$ ii $\cos \theta = \pm \frac{1}{\sqrt{6}}$

10 a $\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

b $\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

11 Let $A = B = x$,

Then $\sin 2x = \sin x \cos x + \sin x \cos x$
 $= 2 \sin x \cos x$

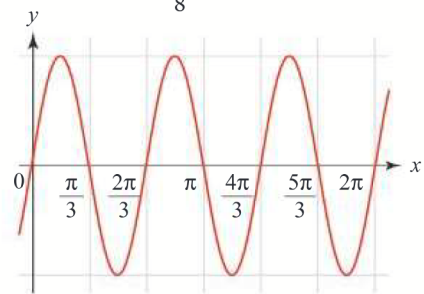
12 $10 \sin(\theta + 53.1)$

13 a 0.05

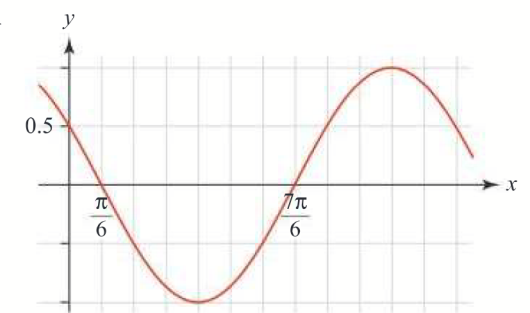
b $\frac{7}{8}$

c 0.5

14 a i



ii



b $x = \frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}$

15 a 13.2 cm, 5.25 cm b 60.3 cm²

16 50.4 cm

17 a 6500

b 10600

c 3 weeks 3 days or 24 days

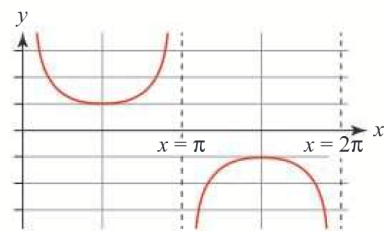
18 a i 4.9 cm²

ii 9.8 cm

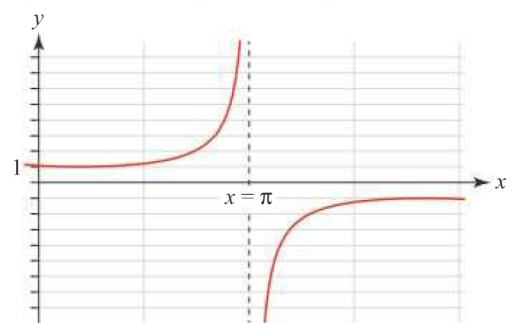
b i 0.51 cm²

ii 5.53 cm

19 a i



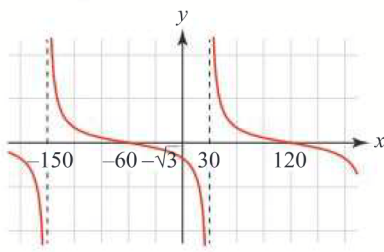
ii



b i $y \in \mathbb{R}, y \leq -2, y \geq 2$

ii $y \in \mathbb{R}, y \leq -1, y \geq 1$

20 a

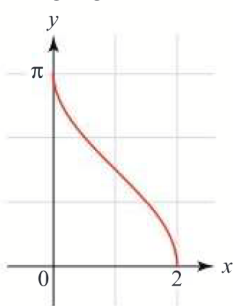


b $x = 30, x = -150$
 $x = 108.7^\circ, -71.3^\circ$

21 i $x = \frac{2\pi}{3}, \frac{4\pi}{3}$

ii $x = \frac{3\pi}{8}, \frac{7\pi}{8}$

22 a



b $y \in \mathbb{R}, 0 \leq y \leq \pi$
c $f^{-1}(x) = 1 + \cos x$
Domain is $0 \leq x \leq \pi$
Range is $0 \leq y \leq 2$

23 a $x = \frac{1+\sqrt{3}}{2}$ b $x = 5 - \frac{\sqrt{3}}{3}$

24 a $2(\sec^2 x - 1) = \sec x - 1$
 $2\sec^2 x - 2 = \sec x - 1$
 $2\sec^2 x - \sec x - 1 = 0$

b $x = 0, 2\pi$

25 a $\frac{\sin^2 \theta}{\sin^2 \theta} + \frac{\cos^2 \theta}{\sin^2 \theta} \equiv \frac{1}{\sin^2 \theta}$
 $1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta$
 $\operatorname{cosec}^2 \theta - \cot^2 \theta \equiv 1$
b $\theta = 18.4^\circ, 198.4^\circ$

26 a $\sec^4 x - \tan^4 x$
 $= (\sec^2 x - \tan^2 x)(\sec^2 x + \tan^2 x)$
 $= \sec^2 x + \tan^2 x$

b $x = \pm 1.11 \text{ rad}, \pm 2.03 \text{ rad}$

27 a $\cos 3x = \cos(2x + x)$
 $= \cos 2x \cos x - \sin x \sin 2x$
 $= (2\cos^2 x - 1)\cos x - 2\sin x(2\sin x \cos x)$
 $= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$
 $= 2\cos^3 x - \cos x - 2(1 - \cos^2 x)\cos x$
 $= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$
 $= 4\cos^3 x - 3\cos x$

b $x = \frac{\pi}{18}, \frac{11\pi}{18}, \frac{13\pi}{18}, \frac{23\pi}{18}, \frac{25\pi}{18}, \frac{35\pi}{18}$

28 a $\cos 2x = \cos x \cos x - \sin x \sin x$
 $= \cos^2 x - \sin^2 x$
 $= \cos^2 x - (1 - \cos^2 x)$
 $= 2\cos^2 x - 1$

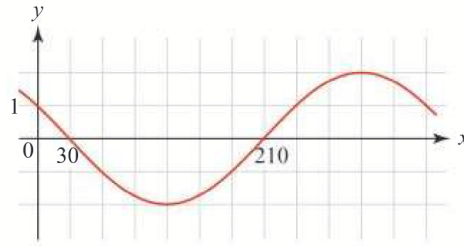
b $x = 120^\circ, 180^\circ, 240^\circ$

29 e.g. $A = 30^\circ, B = 60^\circ$
 $\cos(30 + 60) = \cos 90 = 0$
 $\cos 30 + \cos 60 = \frac{\sqrt{3}}{2} + \frac{1}{2} \neq 0$
Therefore, in general,
 $\cos(A + B) \neq \cos A + \cos B$

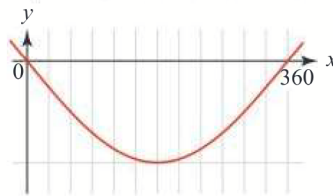
30 a $f(x) = 4\sqrt{5}\cos(x - 0.464)$

b $x = 1.78 \text{ rad}, 5.43 \text{ rad}$

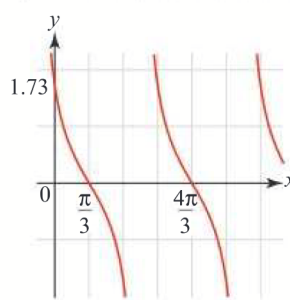
31 a



b



32 a



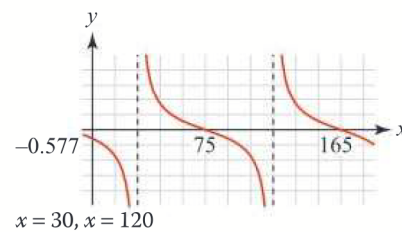
b $x = \frac{5\pi}{6}, x = \frac{11\pi}{6}, x = \frac{\pi}{6}, \frac{7\pi}{6}$

33 16.8 cm^2

34 a 10.7 cm

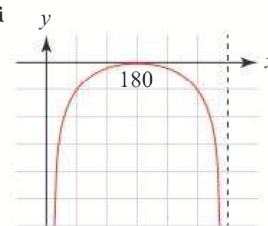
b i 59.2 cm^2 ii 33.7 cm

35 a i



$x = 30, x = 120$

ii



b $x = 0, x = 360$

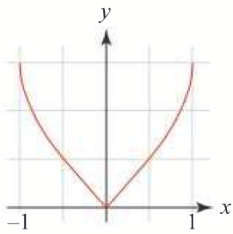
$x = 8.3^\circ, 98.3^\circ, 128.3^\circ$

36 a $\theta = 0.421, 3.56, 2.72, 5.86 \text{ rad}$

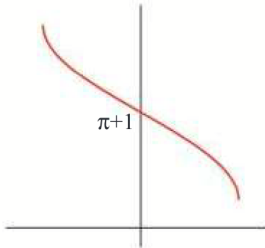
b $\theta = \frac{\pi}{12}, \frac{19\pi}{12}, \frac{7\pi}{12}, \frac{13\pi}{12}$

37 $x = 90^\circ, 270^\circ, y = 1, -1$

38 a



b



$$39 \frac{5^2 \sec^2 t}{25} - \frac{3^2 \tan^2 t}{9} = \sec^2 t - \tan^2 t = 1$$

$$40 x = 36.9^\circ, 216.9^\circ, 135^\circ, 315^\circ$$

$$41 \theta = 0.421, 2.721, \frac{\pi}{3}, \frac{2\pi}{3} \text{ rad}$$

$$42 \text{ LHS} = \frac{\cos^2 \theta}{\sin \theta} - \sin \theta = \frac{\cos^2 \theta - \sin^2 \theta}{\sin \theta} = \frac{1 - 2 \sin^2 \theta}{\sin \theta} = \csc \theta - 2 \sin \theta$$

$$43 x < \frac{\pi}{6}, x > \frac{5\pi}{6}$$

$$44 \text{ a i LHS} = \frac{\cos x(1 - \cos x) - \sin x \sin x}{\sin x(1 - \cos x)} = \frac{\cos x - \cos^2 x - \sin^2 x}{\sin x(1 - \cos x)}$$

$$= \frac{\cos x - 1}{\sin x(1 - \cos x)} = \frac{-1}{\sin x} = -\csc x \text{ as required}$$

$$\text{ii } x \neq 2n\pi$$

$$\text{b } x = 3.48, 5.94 \text{ rad}$$

$$45 \text{ a } \cos x = \cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{2}\right) - \sin\left(\frac{x}{2}\right)\sin\left(\frac{x}{2}\right) = \cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) = 1 - \sin^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right) = 1 - 2\sin^2\left(\frac{x}{2}\right) \text{ as required}$$

$$\text{b } x = 250.5^\circ, 109.5^\circ$$

$$46 \text{ Earliest time } 15:04$$

$$\text{Latest time } 20:32$$

$$47 x = 120^\circ, 180^\circ, 240^\circ$$

$$48 \text{ a } r = \sqrt{5} \text{ or } 2.24$$

$$\alpha = 63.4$$

$$\text{b Minimum is } \frac{1}{5}$$

$$\theta = 26.6^\circ$$

$$\text{c Maximum is } \frac{1}{3 - \sqrt{5}}$$

$$\theta = 206.6^\circ$$

